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Fatigue crack growth with overload under spectrum loading

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Abstract

Load cycle interactions can have a very significant effect in fatigue crack growth under variable amplitude loading. Studying of fatigue crack growth rate and fatigue life calculation under spectrum loading is very important for the reliable life prediction of engineering structures. In this paper, a fatigue life prediction model under various load spectra, using the strain energy density factor approach and the plastic zone size near crack tip as main parameters in calculating effective strain energy density factor, has been proposed. The present model is validated with fatigue crack growth test data provided by Ray under various variable amplitude and spectrum loading in 7075-T6 and 2024-T3 aluminum alloy, respectively. Predictions of present model are compared with those of the state-space model, FASTRN and AFGROW codes. The results show that the predicted results agree well with the test data.

Keywords: Spectrum loading; Crack opening ratio; Effective SEDF range; Fatigue life prediction

1. Introduction

Many engineering structures are subjected to random loading in service. The fatigue growth life will be affected by load sequence. Neglecting the effect of cycle interaction in fatigue calculations under variable amplitude loading can lead to completely invalid life predictions. However, for design purposes it is particularly difficult to generate an algorithm to quantify these sequence effects on fatigue crack propagation, due to the number and to the complexity of the mechanisms involved in this problem [1]. One of the theories to explain these load sequence effect is that the plasticity induced fatigue crack closure is the primary mechanism [2]. There are many calculating models of crack propagation life under spectrum loading, such as Wheeler model, Willenborg et al. model, based on plastic zone correction theory in the vicinity of crack tip [3–5], the $U \sim R$ model based on the concept of effective stress intensity factor range [6–13] and the model based on strain energy density factor [14–16].

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In this paper, a feasible study towards the crack propagation law under various spectra loading has been carried out based on effective strain energy density factor. A crack growth life prediction model under spectrum loading is provided. Predictions of present model are compared with test data, those of the state-space model, FASTRN and AFGROW codes under various variable amplitude and spectrum loading.

2. Fatigue crack growth under spectrum loading

2.1. Fatigue crack propagation rate

It is difficult to give a crack growth life calculation model which can consider all the effect factors such as plastic induced closure, residual stresses and strain, strain hardening, crack face roughness, and oxidation of the crack face, et al. The results of experimental study on fatigue over the past several decades provide a knowledge base, and the primary mechanism under many conditions is plasticity. The purpose of this paper is to address how to characterize the effect of load sequence in fatigue crack propagation under variable amplitude loading.

The fatigue crack propagation will be decreased or arrested after experiencing overload one or more times. This phenomenon can be explained by crack opening ratio U. When it is overloaded, the increased $\sigma_{\max}$ and unchanged $\sigma_{\min}$ lead to the decrease of stress ratio $R = \sigma_{\min}/\sigma_{\max}$ and U is accordingly decreased. When it is underloaded, stress ratio R changes similarly while U proves to increase unexpectedly [10]. How to take account of the effect during the load cycles after overloading and underloading is the main problem.

Paris crack growth law is widely used to calculate the fatigue crack propagation life in engineering structures. The expression may not be adequate to analyze the crack growth behavior of cracked structures under spectrum loading for the equation dose not involve the mean stress level and the equation is restricted to cracks propagated normal to the applied load [16]. The strain energy density factor (SEDF) approach has been used to analyze fatigue crack growth behavior of cracked structures [14,17]. After the effect of load sequence was discussed by Schijive and Broek and the effective stress intensity factor was proposed by Elber [5], the crack fatigue growth rate was expressed $\mathrm{d}a/\mathrm{d}N$ versus ΔK_{eff}. A number of load-interaction model have been developed to correlate fatigue crack growth rates and to predict crack growth under variable amplitude loading during the past three decades. In this paper, the effective strain energy density factor range is used in the crack growth rate equation. The crack growth rate can be expressed as:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = B(\Delta S_{\mathrm{eff}})^m \tag{1}$$

$$\Delta S_{\mathrm{eff}} = C \cdot \Delta S \tag{2}$$

where $\mathrm{d}a/\mathrm{d}N$ is the crack growth rate, ΔS is the strain energy density factor (SEDF) range, ΔS_{eff} is the effective strain energy density factor range, B and m are material constants, C is a correction coefficient of plastic zone size in the vicinity of crack tip.

2.2. Strain energy density factor

The strain energy density factor S takes the following form:

$$S = a_{11}K_1^2 + 2a_{12}K_1K_2 + a_{22}K_2^2 + a_{33}K_3^2 \tag{3}$$

in which K_1, K_2 and K_3 are the stress intensity factors (SIF) to tensile, in-plane shear, and out-of-plane shear loads, a_{ij} $(i, j = 1, 2, 3)$ are the coefficients. In this study, $K_2 = K_3 = 0$ and the direction of the crack growth is found by taking $\partial S/\partial\theta = 0$ which gives $\theta = 0$. The strain energy density factor S in the case of plane strain is given by

$$S = [(1 - 2v)/4G] \cdot K_1^2 \tag{4}$$

The strain energy density factor range is given by

$$\Delta S = \frac{1 - 2v}{4G}(K_{1,\max}^2 - K_{1,\min}^2) = \frac{1 - 2v}{4G}(K_{1,\max} + K_{1,\min})\Delta K_1 \tag{5}$$

where v is Poisson's ratio and G is the shear modulus.

2.3. Calculation of the effective strain energy density factor range

Investigations indicate that crack opening ratio can also be affected by maximum stress intensity factor and the constrained state of the crack tip. Meggiolaro pointed out that the crack closure concept dominated by stress ratio cannot provide rational explanation on certain conditions (e.g. crack propagation retardation or arrest under high stress ratio) [1]. Thus, in this model, a correction coefficient of plastic zone size in the vicinity of crack tip due to overloading or underload is introduced. The effective strain energy density factor range is expressed as

Table 1
Geometry parameters and material properties of the specimens

Specimen material	σ_y (MPa)	σ_u (MPa)	B	m	n	t (mm)	w (mm)
7075-T6 aluminum alloy	520	575	9.9×10^{-4}	2.07	0.5	4.1	305
2024-T3 aluminum alloy	327.9	473.3	5.08×10^{-4}	2.04	0.5	4.1	229

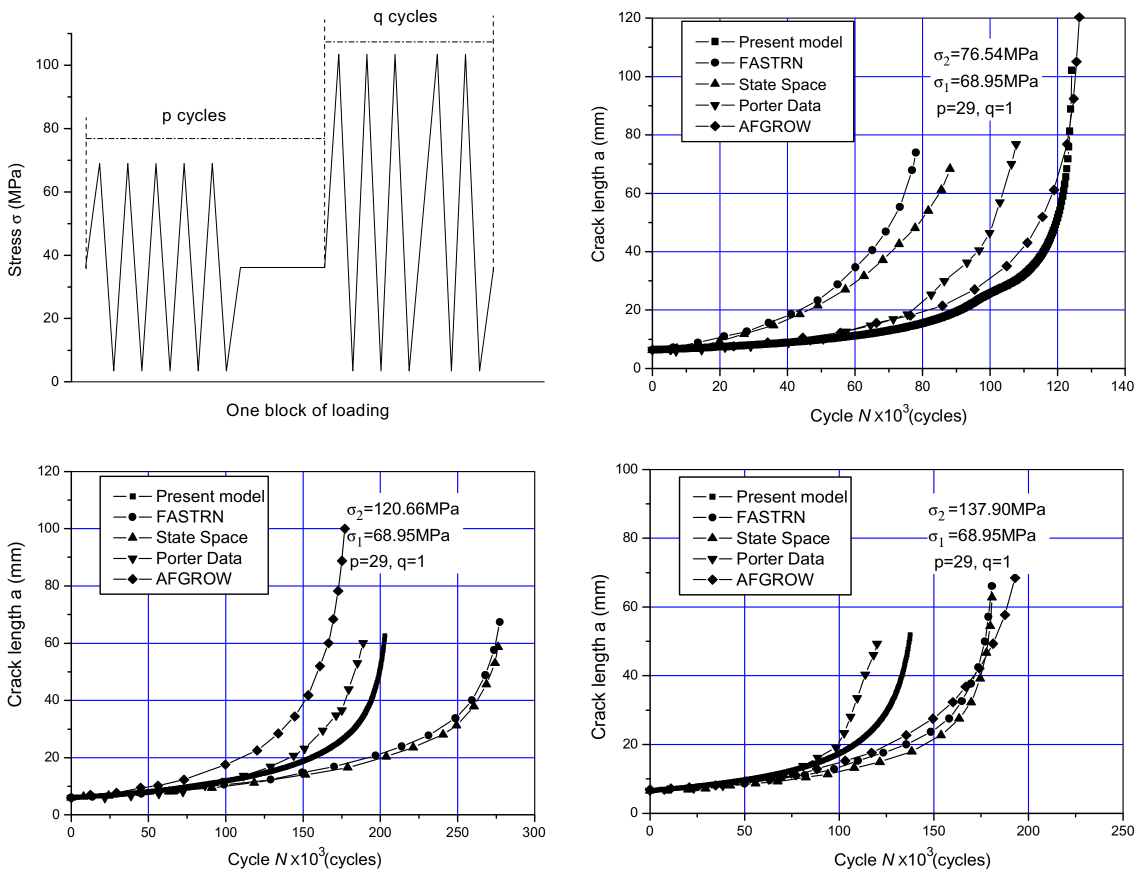


Fig. 1. Comparison of predicted values with test data and those of state-space, FASTRN and AFGROW codes (at different overload ratios).

$$\Delta S_{\text{eff}} = C \cdot \Delta S = C\frac{1-2v}{4G}(K_{1,\max}^2 - K_{1,\min}^2)$$
$$= \frac{1-2v}{4G}(K_{1,\max} + K_{1,\min})C\Delta K_1 \qquad (6)$$

Lots of research works on retardation response of overloading, based on the correction of yield zone vicinity of crack tip, have been reported [3]. A fatigue life prediction model, which considering the constrain states of the crack tip depend on the applied stress, yield strength of material and the specimen thickness, under spectrum loading was reported in literature [4]. In this paper, the model in literature [5] is employed and modified that the effect of underload is included in the present model. A physical explanation of crack retardation due to enlarged plastic zone is presented below.

The plastic zone size is relatively small under constant amplitude loading, while the resulting plastic zone becomes larger when a single overload is applied. Provided that crack length is a (a_{OL}) and the applied stress is $\sigma_{\max}$ ($\sigma^{\bullet}_{\max}$) corresponding present load and (overload) respectively. A plastic zone with size of r_y ($r^{\bullet}_{\text{OL}}$) will appear in the vicinity of crack tip. The size of plastic zone, r_y, can be calculated using Eqs. (8) and (10) which considering plate thickness and yield strength given by Voorwald et al. [4]. Wheeler assumes that overload response will effect if only $a + r_y$ does not exceed the range of $a_{\text{OL}} + r^{\bullet}_{\text{OL}}$ during the stress cycles after experiencing an overload. The overload response will disappear when $a + r_y$ reaches to or exceed $a_{\text{OL}} + r^{\bullet}_{\text{OL}}$ due to crack propagation or stress increase.

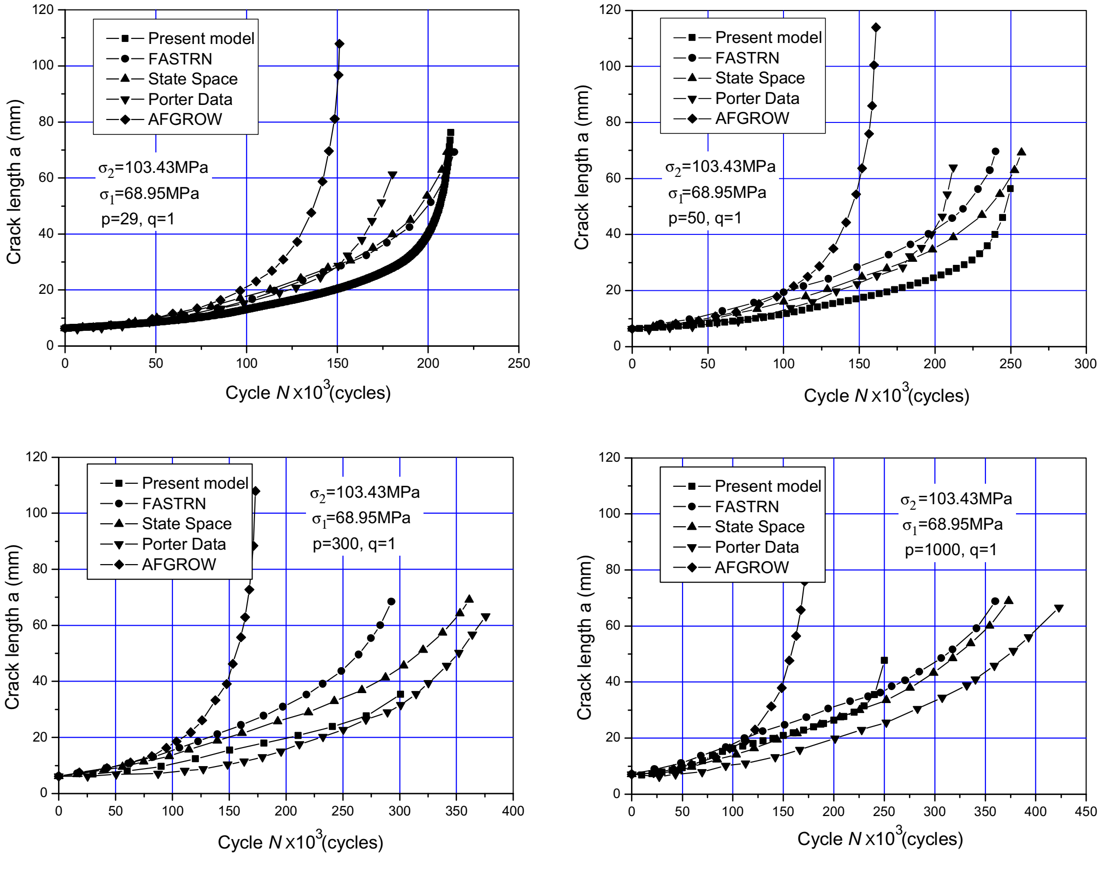


Fig. 2. Comparison of predicted values with test data and those of state-space, FASTRN and AFGROW codes (at different p).

In the contrast to a single-cycle overload, a single-cycle underload makes the reverse plastic flow and depletion of the resulting plastic zone. The increment of yield zone size caused by underload can be quantitively calculated by Eq. (9). The correction coefficient of plastic zone size can be calculated using the following expressions:

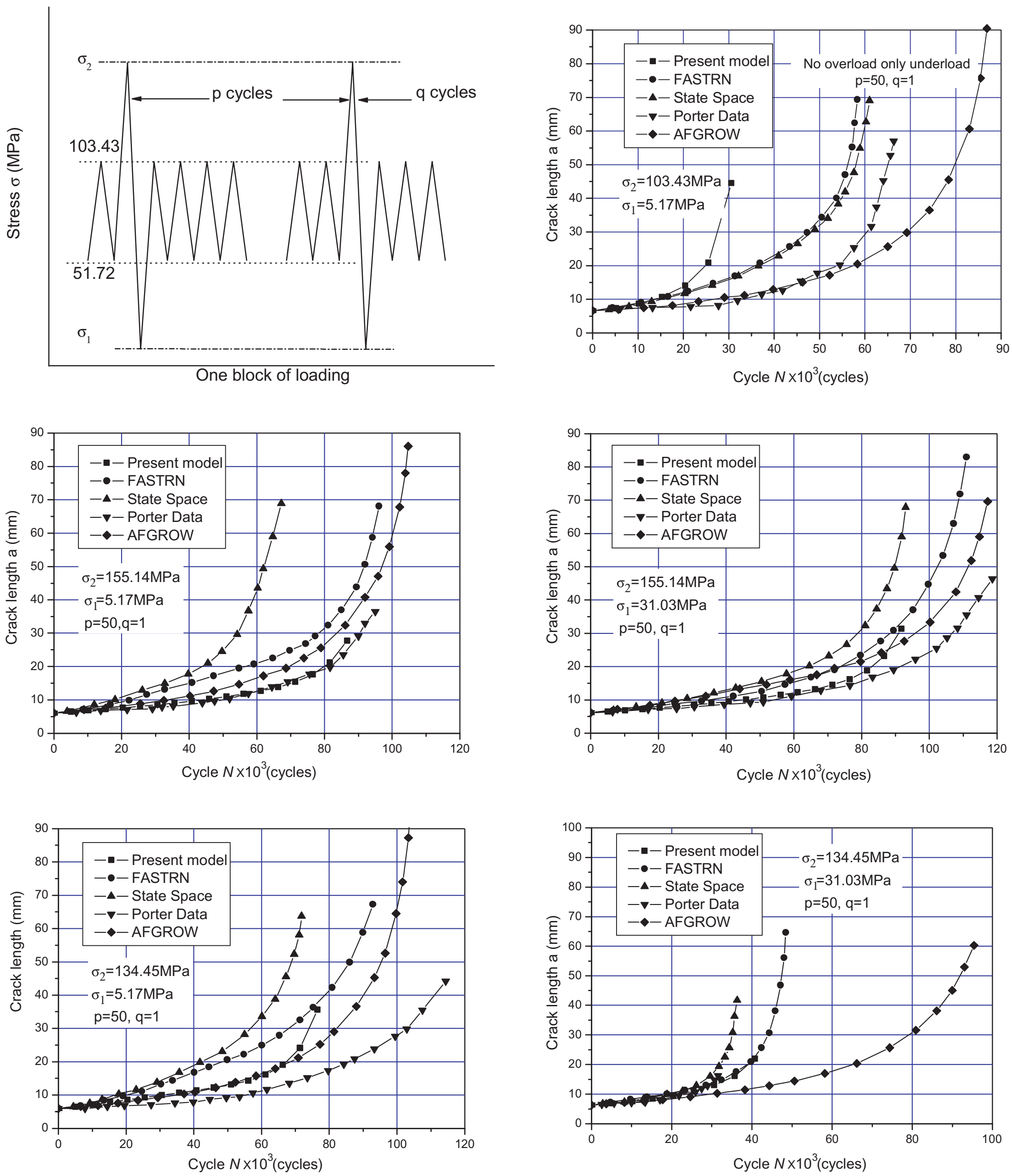


Fig. 3. Comparison of predicted values with test data and those of state-space, FASTRN and AFGROW codes (overload–underload at different load ratios).

$$C=\begin{cases}\left(\dfrac{r_y}{a_{OL}+r^{\bullet}_{OL}-a-r_{\Delta}}\right)^n & (a+r_y<a_{OL}+r^{\bullet}_{OL}-r_{\Delta})\\ 1 & (a+r_y\geqslant a_{OL}+r^{\bullet}_{OL}-r_{\Delta})\end{cases} \tag{7}$$

$$r_y=\beta\left(\frac{K_{max}}{\sigma_y}\right)^2 \tag{8}$$

$$r_{\Delta}=\beta\left(\frac{\Delta K_{max}}{\sigma_y^-}\right)^2 \tag{9}$$

$$\beta=\begin{cases}\dfrac{1}{6\pi} & (t\geqslant 2.5(K_{max}/\sigma_y)^2)\\ \dfrac{1}{\pi} & \left(t\leqslant\dfrac{1}{\pi}(K_{max}/\sigma_y)^2\right)\\ \dfrac{1}{6\pi}+\dfrac{5}{6\pi}\left(\dfrac{2.5-t(K_{max}/\sigma_y)^{-2}}{2.5-\pi^{-1}}\right) & \left(\dfrac{1}{\pi}\left(\dfrac{K_{max}}{\sigma_y}\right)^2<t<2.5\left(\dfrac{K_{max}}{\sigma_y}\right)^2\right)\end{cases} \tag{10}$$

$$\Delta K_u=M\sqrt{\pi a}(\sigma^{i-1}_{min}-\sigma^{i}_{min}) \tag{11}$$

where a, a_{OL} are crack length at present and at prior overloading, r_y, r^*_{OL} are yield zone size under present maximum stress and under maximum stress of prior overloading, r_{Δ} is increment of yield zone size caused by underloading, t is plate thickness, σ_y, σ_y^- are tensile, compressive yield stress, K_{max} is maximum stress intensity factor in every load cycle, σ^{i-1}_{min}, σ^{i}_{min} are minimum stresses in load cycle $i-1$ and i, n is material constant determined by test. $n=0$ when load sequence effect can be neglected.

Substituting $K^{\bullet}_{max}$ and K_{max} corresponding to the prior overload and present load respectively into Eq. (8), $r^{\bullet}_{OL}$ and r_y are determined. Parameter r_{Δ} is used to consider the effect of underload.

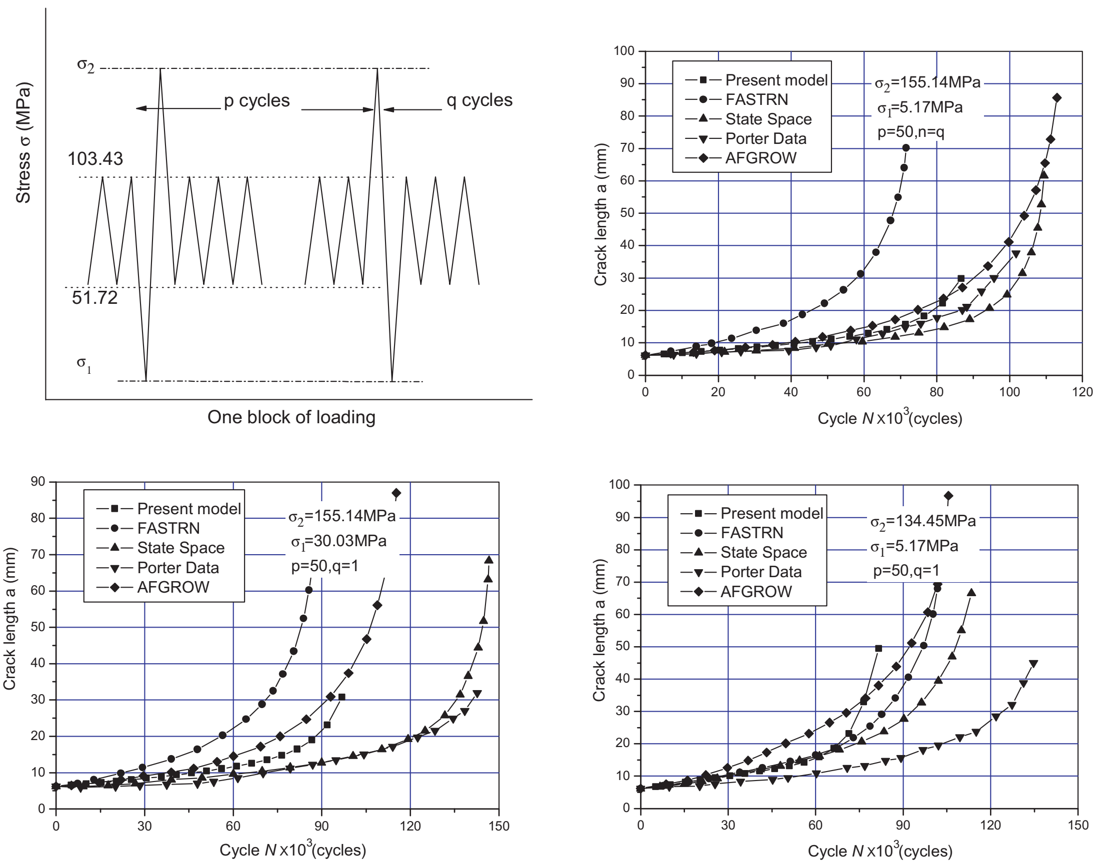
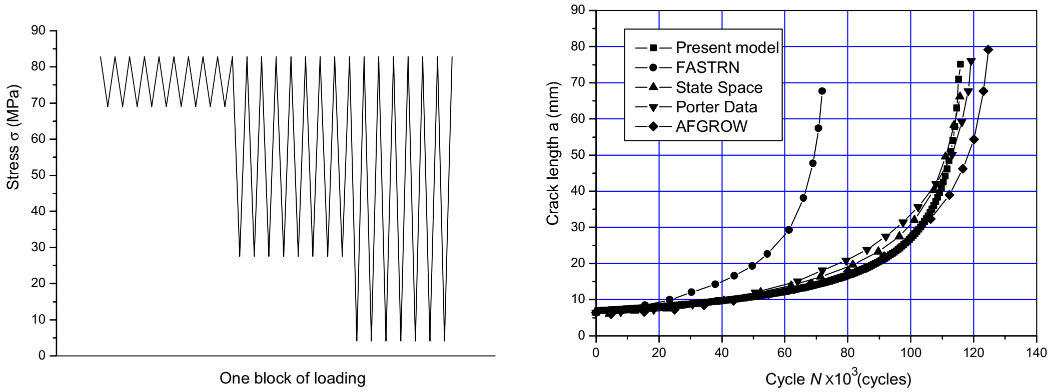
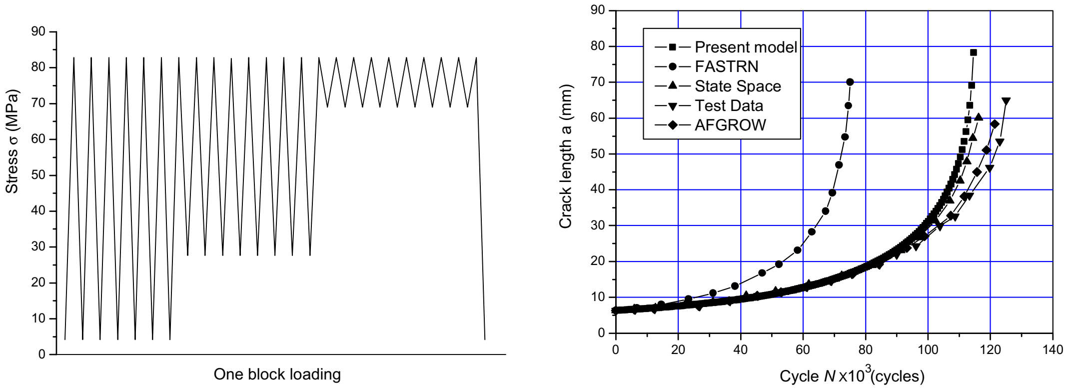


Fig. 4. Comparison of predicted values with test data and those of state-space, FASTRN and AFGROW codes (underload–overload at different load ratios).

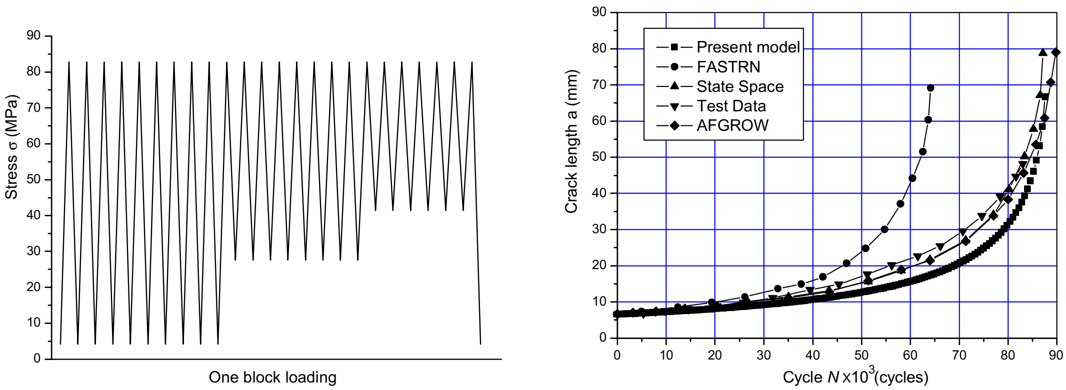
From present model and the correlative parameters, the overloading retardation responses can be expressed by above equations. These effect parameters including maximum and



Program P1

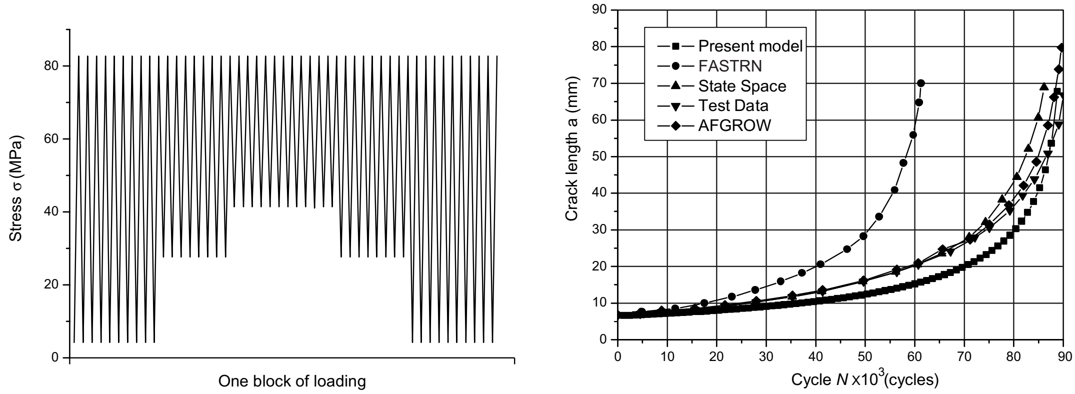


Program P2

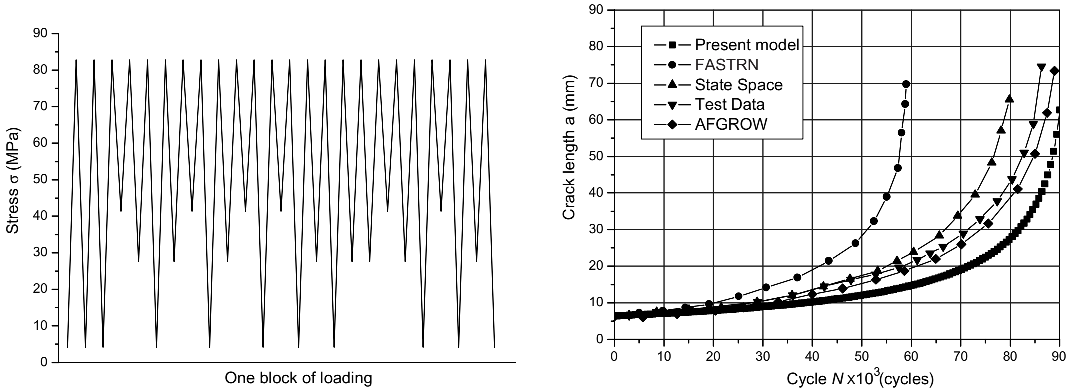


Program P3

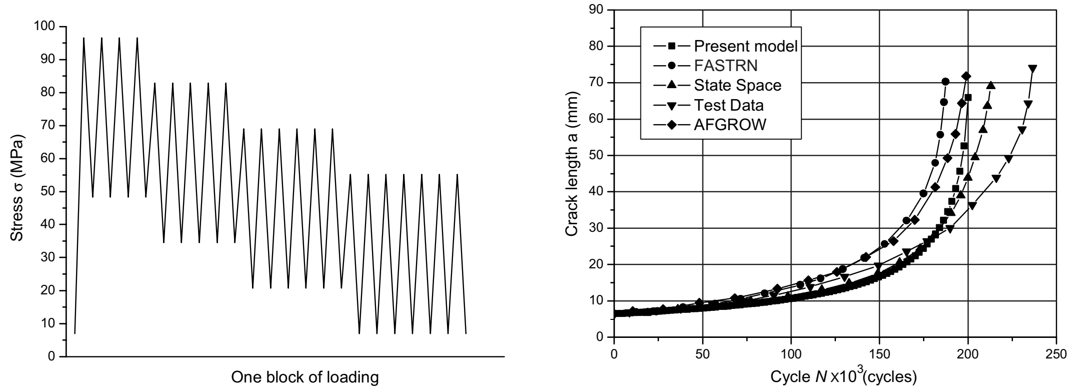
Fig. 5. Comparison of predicted values with test data and those of state-space, FASTRN and AFGROW codes under six types of block loading.



Program P4



Program P5



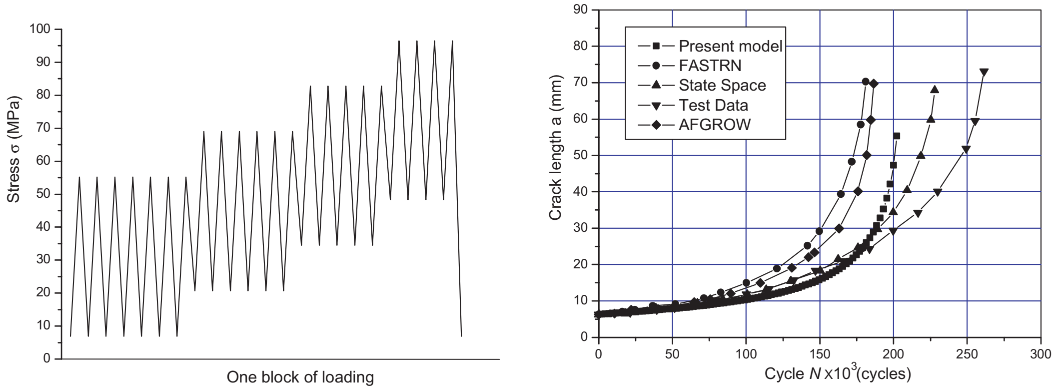
Program P6

Fig. 5 (*continued*)

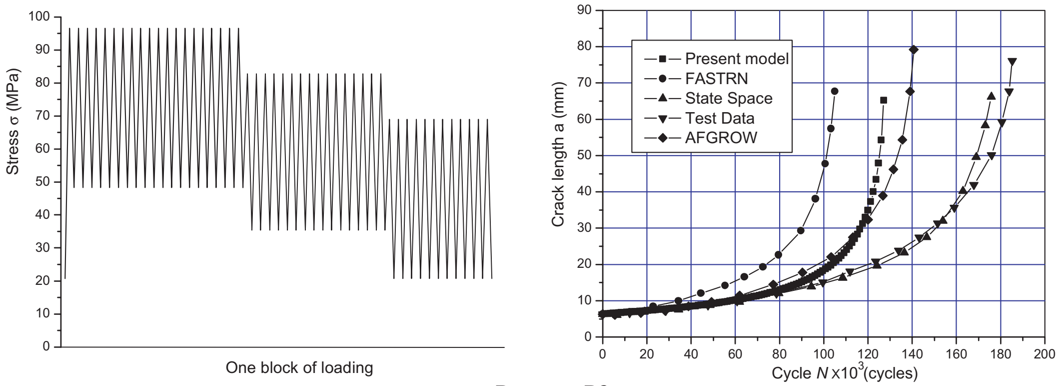
minimum stresses of a cycle, yield strength of material and the plate thickness are considered in the effective stress intensity factor range calculation.

3. Model validation

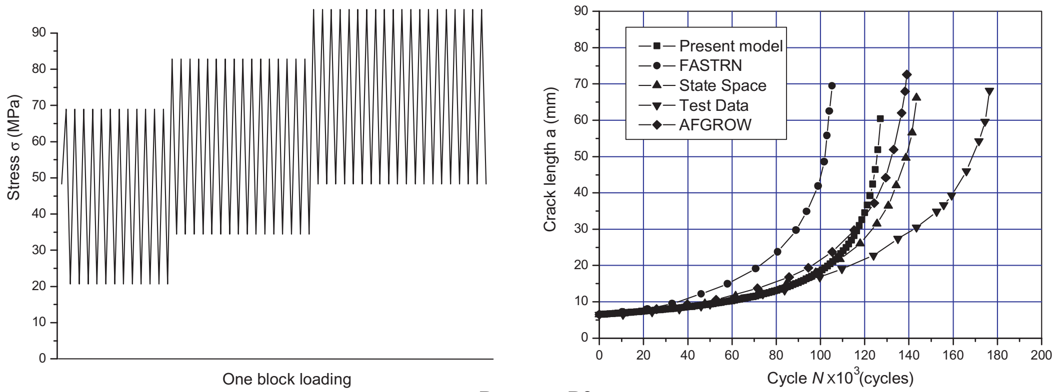
The fatigue crack propagation model under variable amplitude loading is presented above.



Program P7



Program P8

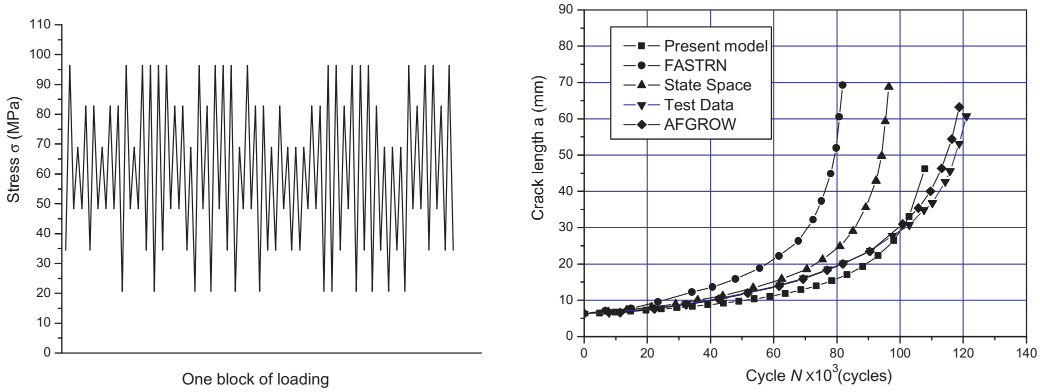


Program P9

Fig. 5 (*continued*)

For demonstrating the validation of the model, predictions of present model are compared with test data, those of the state-space model, FASTRN and AFGROW codes given in Ref. [11].

Porter provided some fatigue test data of through thickness center-cracked 7075-T6 aluminum alloy plate specimens under variable amplitude loading. The geometry parameters and



Program P10

Fig. 5 (*continued*)

material properties of the specimens are listed in Table 1. Fig. 1 illustrates the cyclic stress excitation and the comparisons of predicted values with Porter data and those of the state-space model, FASTRN and AFGROW codes under different overload ratios ($\sigma_2/\sigma_1 = 76.54/68.95$, 120.66/68.95 and 137.9/68.95 respectively). Present model can give better prediction at high stress ratio. Fig. 2 shows comparisons of predicted values with Porter data, those of the state-space model, FASTRN and AFGROW codes, under different number of p ($q = 1$, $p = 29$, 50, 300 and 1000 respectively). Fig. 3 shows comparisons of predicted values with test data and those of state-space, FASTRN and AFGROW codes under overload–underload at different load ratios. Fig. 4 shows comparisons of predicted values with test data and those of state-space, FASTRN and AFGROW codes under underload–overload at different load ratios. From these comparisons, it can be seen that the predicted values are in good agreement with test data.

The model has also been used to predict the fatigue test results on center-cracked 2024-T3 aluminum alloy specimens under program loading given by Ray [11]. The geometry parameters and material properties of the specimens are presented in Table 1. The load spectra and the schematic comparisons of predicted values with test data and those of state-space, FASTRN and AFGROW codes are presented in Fig. 5. It can be seen from the comparisons that the predicted values under 10 different program loadings agree well with test data. It is clear that neglecting the effect of load sequence in fatigue calculations under variable amplitude loading can lead to completely invalid life predictions. The present model has been proved to be rationally applicable for crack propagation under variable amplitude loading.

4. Summary and conclusions

Fatigue crack closure is the most used mechanism to explain load cycle interactions such as delays in or arrests of the crack growth after overloads. In fact, neglecting crack closure in many fatigue life calculations can result in overly conservative predictions, increasing maintenance costs by unnecessarily reducing the period between inspections. Many models based on effective stress intensity factor take the stress ratio as the main effect factor in considering the effect of load sequence. It has long been proved to satisfactorily explain plane stress crack retardation effects. But in the procedure based on SEDF, the effect of mean stress is included in SEDF. For better predicting the delay of the overload, a correction coefficient of plastic zone size in the vicinity of crack tip is introduced.

A crack growth life calculation model based on effective strain energy density factor under variable amplitude loading is presented in this paper. In this model, the effect of mean stress is included in strain energy density factor and the overload retardation is corrected by the plastic zone size in the vicinity of crack tip. Thus the model has distinct advantage to deal with problem of overload response and is appropriate to characterize crack propagation under variable loading. The model is used to predict the fatigue crack propagation test data and compared with those of state-space, FASTRN and AFGROW codes under several types of spectrum loadings. The comparisons show that the present model has satisfactory accuracy.

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