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Nonlinear hydroelastic analysis of a moored floating body☆

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Abstract

By integration of the second-order fluid pressure over the instantaneous wetted surface, the generalized first- and second-order fluid forces used in nonlinear hydroelastic analysis are obtained. The expressions for coefficients of the generalized first- and second-order hydrodynamic forces in irregular waves are also given. The coefficients of the restoring forces of a mooring system acting on a flexible floating body are presented. The linear and nonlinear three-dimensional hydroelastic equations of motion of a moored floating body in frequency domain are established. These equations include the second-order forces, induced by the rigid body rotations of large amplitudes in high waves, the variation of the instantaneous wetted surface and the coupling of the first order wave potentials. The first-order and second-order principal coordinates of the hydrelastic vibration of a moored floating body are calculated. The frequency characteristics of the principal coordinates are discussed. The numerical results indicate that the rigid resonance and the coupling resonance of a moored floating body can occur in low frequency domain while the flexible resonance can occur in high frequency domain. The hydroelastic responses of a moored box-type barge are also given in this paper. The effects of the second-order forces on the modes are investigated in detail.

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Nomenclature

$\vec{a}$	a position vector defined in the system of $O'x'y'z'$;
a_{rk}	elements of the generalized mass matrix of the structure;
a_{rr}	diagonal elements of the generalized mass matrix;
$\vec{A}$	a position vector defined in the frame $Oxyz$;
$A_{rk}(\omega_j)$	coefficients of added mass;
$A_{rr}(\omega)$	diagonal elements of the added mass matrix;
b_{rk}	elements of the generalized damping matrix of the structure;
$B_{rk}(\omega_j)$	coefficients of added damping;
c_{rk}	elements of the generalized rigid matrix of the structure;
c_{rr}	diagonal elements of the generalized rigid matrix;
C_W	waterline;
C_{rk}	frequency independent coefficients of the generalized first-order force;
C_{rr}	diagonal elements of the generalized restoring matrix;
Cm_{rr}	diagonal elements of the generalized mooring force restoring matrix;
ΔC_{rk}	first order restoring coefficients;
Cm^j_{rk}	elements of the restoring force matrix of the j-th mooring line;
$D^{(2)}_r(t)$	generalized second-order radiation forces;
D_{rij}	sum frequency forces;
E'	Young's modulus of the mooring line;
$E^{(1)}_r(t)$	generalized first-order wave exciting forces;
$E^{(2)}_r(t)$	generalized second-order forces related to the radiation wave velocity and the incident-diffracted wave velocity;
$F^{(2)}_r(t)$	generalized second-order forces related to the incident and diffracted wave potentials;
g	gravity acceleration;
$H^{(1)}_r(t)$	generalized first-order radiation forces;
$\mathbf{H}$	second-order component of the transformation matrix $\mathbf{T}$;
$\mathbf{I}$	zero-order component of the transformation matrix $\mathbf{T}$;
J	number of the mooring lines;
k_j	wave number of the j-th components;
L	length of the mooring;

$\mathbf{l}$	transformation matrix between the local coordinate system of the structural element and the whole coordinate system of the floating body;
$\mathbf{L}$	a unit matrix;
m	number of the modes;
$M(t)$	vertical bending moment;
M_r	modes of the bending moment;
$\vec{n} = (n_1,n_2,n_3)$	normal vectors of the body's wetted surface defined in $Oxyz$;
N	number of the regular wave components;
$\vec{N} = (N_1,N_2,N_3)$	normal vectors of the body's wetted surface defined in $O'x'y'z'$;
$\mathbf{N}$	embedding matrix of the structural element;
$Oxyz$	equilibrium frame coordinate system;
$O'x'y'z'$	body fixed axes system;
$p(x,y,z,t)$	pressure of the fluid;
$p^{(1)}$	first-order pressure of the fluid;
$p^{(2)}$	second-order pressure of the fluid;
p_r	complex amplitude of the principal coordinate;
$p_r(t)$	principal coordinates;
p_k	steady components of the principal coordinates;
$p_k(\omega_{ij})$	difference frequency components of the principal coordinates;
$p_k(\omega_{ij})$	sum frequency components of the principal coordinates;
Q_r	generalized gravity forces;
Q_{rij}	difference frequency forces;
$R_r^{(1)}(t)$	generalized first-order restoring forces;
$\Delta R_r(t)$	first order forces induced by the rotation of the rigid body modes;
$\mathbf{R}$	first-order component of the transformation matrix $\mathbf{T}$;
s	section area of the mooring line;
$\bar{S}$	mean wetted surface;
$S(t)$	instantaneous wetted surface;
ΔS	difference of the instantaneous wetted surface and the mean wetted surface;
$S_r^{(2)}(t)$	generalized second-order steady forces;
$\mathbf{T}$	transformation matrix of the coordinate systems;
$\vec{u}$	total displacement described in the axes system $Oxyz$;
u_1,u_2,u_3	surge, sway, and heave motions of the body respectively;
$\vec{u}_r$	distortion of the body in its r-th principal dry mode;
$\vec{u}_D$	distortion of the body defined in the body-fixed coordinate system $O'x'y'z'$;

$\overrightarrow{u}_r^0$	r-th mode of the structure in vacuum;
$\overrightarrow{u}_{rj}^0$	displacement modes of the mooring point of the j-th mooring line;
$\overrightarrow{u}^{(1)}$	first order displacement;
U_{ej}	modes of the structural element where the j-th mooring point line belongs to;
$\overrightarrow{v}(x,y,z,t)$	velocity vector of the fluid;
$V(t)$	vertical shearing force;
V_r	shearing force modes;
$w(t)$	vertical displacement;
w_r	dynamic displacement modes;
X	arbitrary chosen complex function or quantity;
Y	arbitrary chosen complex function or quantity;
$\tilde{z}$	relative vertical displacement of the body and the wave surface;
$Z_r(t)$	generalized fluid forces;
$Z_r^{(0)}$	generalized constant forces;
$Z_r^{(1)}(t)$	generalized first-order forces;
$Z_r^{(2)}(t)$	generalized second-order forces;
$\Delta Z_r^{(2)}(t)$	generalized second-order forces induced by the variation of the instantaneous wetted surface;
ρ	density of the fluid;
α	angle between the horizontal and the tangent directions of the mooring line;
β	angle between the horizontal projection of the mooring line and the x direction;
$\theta_1,\theta_2,\theta_3$	roll, pitch, and yaw of the body respectively;
ω	wave circular frequency;
ω_j	wave circular frequency of the j-th components;
ω_{ij}	difference frequency;
ω_{ij}	sum frequency;
ω_{wr}	wet resonant frequency;
ζ_j	wave amplitude of the j-th components;
$\zeta_r(\omega_j)$	coefficients of the generalized first-order wave exciting forces;
ζ^*	wave elevation on the waterline;
$\overrightarrow{\eta}_T$	translation of the $O'x'y'z'$ system with respect to the reference $Oxyz$;
$\overrightarrow{\eta}_R$	rotation of the body with respect to the $O'x'y'z'$ system;
ε	parameter of the slenderness;
ε_j	random phase angle of the j-th components;

ε_{ij}^{-}	difference phase angle;
ε_{ij}^{+}	sum phase angle;
$\phi(x,y,z,t)$	unsteady velocity potential defined in the equilibrium frame;
$\phi_I(x,y,z,t)$	incident wave potential;
$\phi_D(x,y,z,t)$	diffraction wave potential;
$\phi_r(x,y,z,t)$	radiation wave potential;
$\varphi_I(x,y,z,\omega)$	component of the incident wave velocity potential;
$\varphi_D(x,y,z,\omega)$	component of the diffraction wave velocity potential;
$\varphi_r(x,y,z,\omega)$	components of the radiation wave potential arising from the vibration in the r-th principal dry mode of the flexible body, with unit amplitude and frequency ω;
∇^2	Laplace operator.

1. Introduction

The hydroelasticity theory of floating bodies, which embodies the full complexities of the dynamics of the structure concerned and the fluid around it, provides a more consistent and more rational approach for the assessment of the overall behavior of a flexible marine structure in waves. The existing theories have been developed for more than 20 years, mainly focused on linear problems (Bishop and Price, 1979; Wu, 1984; Price and Wu, 1985; Bishop et al., 1986; Du, 1996; Wang, 1996). The linear theories are suitable for investigating a floating body with a small amplitude of motion or an elastic structure in waves with resonant frequencies greater than the range of the wave frequencies. When responding to high waves with large amplitudes, the floating structure may apparently suffer from non-linear responses due to the second order hydrodynamic forces induced by the rigid body rotations of motion, and the effect of the instantaneous wetted surface. For a marine structure with a scale much larger than a conventional ship, its resonant frequencies in waves will be very low, and its behavior in waves will be quite different from a usual vessel. In these circumstances it is necessary to investigate the influence of the second, and higher order hydrodynamic actions on the hydroelastic responses of a floating structure.

Wu et al. (1997) firstly presented a three-dimensional non-linear hydroelasticity theory. Maeda et al. (1997) and Ikoma et al. (1998) calculated the second-order wave loads acting on the very large floating structures (VLFS). They used the thin plate hydrodynamic theory, and the VLFS was simplified as a floating flexible plate. Chen (2001) presented a second order nonlinear hydroelastic analysis method of a floating system. In this paper, the second-order non-linear hydroelastic analysis method of a moored floating body is presented, and the second-order non-linear hydroelastic equations of a moored floating body are established. Based on a detailed discussion of the frequency characteristics of the first-order principal coordinates of the hydroelastic analysis, the second-order principal coordinates of a moored floating beam model are calculated. The results indicate that the rigid resonance and the coupling resonance of a moored floating body can occur in a low frequency domain, while the flexible resonance can occur in a high frequency domain.

2. Three dimensional potential theory around a flexible floating body

2.1. Basic definitions

The fluid around the moored flexible floating body is assumed to be ideal (i.e. uniform, continuous, inviscid, incompressible and irrotational) and the surface wave is of small amplitude. Hence, the fluid behavior can be fully governed by the velocity potential. In order to simplify the expression, two coordinate systems are introduced, namely the equilibrium frame of *Oxyz*, and the body fixed axes system $O'x'y'z'$, as shown in Fig. 1. The origin of the *Oxyz* system is on the point of the intersection of the still water surface and the vertical line which goes through the gravity center of the structure, and the axis Oz is upward. The $O'x'y'z'$ system is fixed on the floating body.

The displacement at any point of the body is decomposed into three parts: the translation of the $O'x'y'z'$ system $\overrightarrow{\eta}_T$ with respect to the reference frame *Oxyz*, the rotation of the body $\overrightarrow{\eta}_R$ with respect to the $O'x'y'z'$ system, and the distortion of the body $\overrightarrow{u}_D$ defined in the body-fixed coordinate system $O'x'y'z'$. These components may be represented as follows.

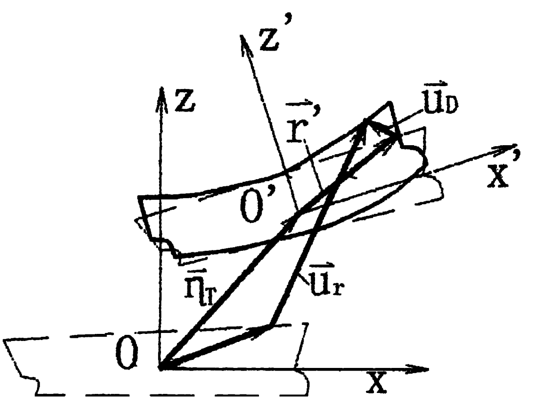


Fig. 1. Definition of coordinate systems.

$$\begin{cases} \vec{\eta}_T = \{u_1,\ u_2,\ u_3\} = \sum_{r=1}^{3} \vec{u}_r, \\ \vec{\eta}_R = \{\theta_4,\ \theta_5,\ \theta_6\} = \sum_{r=4}^{6} \vec{\theta}_r, \\ \vec{u}_D = \sum_{r=7}^{m} \{u_r,\ v_r,\ w_r\} = \sum_{r=7}^{m} \vec{u}_r(x',y',z',t), \end{cases} \tag{1}$$

where u_1,u_2,u_3 represent respectively surge, sway, and heave motions of the body, and $\theta_1,\theta_2,\theta_3$ represent respectively roll, pitch, and yaw. $\vec{u}_r$ denotes the distortion of the body in its r-th principal dry mode. The number of the modes is truncated to m.

Due to the rigid body motion, a position vector $\vec{a}$ defined in the system of $O'x'y'z'$ is represented by a new position vector $\vec{A}$ in the frame $Oxyz$ in the form

$$\vec{A} = \vec{\eta}_T + \mathbf{T}\vec{a}\ , \tag{2}$$

where $\mathbf{T} = \mathbf{I} + \mathbf{R} + \mathbf{H}$ is the matrix of transformation of the coordinate systems, with the accuracy up to the second-order magnitude of θ_r ($r = 4,5,6$). $\mathbf{I}$,$\mathbf{R}$,$\mathbf{H}$ are respectively its zero-, first- and second-order components:

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{R} = \begin{bmatrix} 0 & -\theta_6 & \theta_5 \\ \theta_6 & 0 & -\theta_4 \\ -\theta_5 & \theta_4 & 0 \end{bmatrix}, \tag{3}$$

$$\mathbf{H} = \frac{1}{2}\begin{bmatrix} -(\theta_5^2 + \theta_6^2) & 0 & 0 \\ 2\theta_4\theta_5 & -(\theta_4^2 + \theta_6^2) & 0 \\ 2\theta_4\theta_6 & 2\theta_5\theta_6 & -(\theta_5^2 + \theta_4^2) \end{bmatrix}.$$

Evidently,

$$\mathbf{R}\vec{a} = \vec{\eta}_{\mathrm{R}} \times \vec{a}. \tag{4}$$

When a point $\vec{r}\,'=(x',y',z')$ of the body moves to the position $\vec{r}\,'+\vec{u}_D$ in the frame $O'x'y'z'$ due to distortion $\vec{u}_D$ of the body, its total displacement described in the axes system $Oxyz$ is (see Fig. 1)

$$\vec{u} = \sum_{r=1}^{m} \vec{u}_r + \mathbf{H}\vec{r}\,' + \vec{\eta}_R \times \vec{u}_D + \mathbf{H}\vec{u}_D. \tag{5}$$

In a linear theory where the rotation of the body is small and the linearization is employed, the first order displacement may be represented by only the first term

$$\vec{u}^{(1)} = \sum_{r=1}^{m} \vec{u}_r = \vec{\eta}_T + \vec{\eta}_R \times \vec{r}\,' + \sum_{r=7}^{m} \vec{u}_r. \tag{6}$$

The second term at the right hand side of Eq. (5) is the second order influence of the rigid body rotations to the local displacement. The third term represents the first order influence of the rigid body rotation to the representation of distortion in the equilibrium frame. The last term describes the second order influence of the rigid body rotation to the distortions. For the case of a conventional ship, the four terms at the right hand side of Eq. (5) are of the orders $O(\varepsilon)$,$O(\varepsilon^2)$,$O(\varepsilon^3)$ and $O(\varepsilon^4)$ respectively, where ε is the parameter of the slenderness. For a very large and flexible structure,

$$|\vec{u}_D| = O(\varepsilon),$$

and the four terms may have the orders of $O(\varepsilon)$, $O(\varepsilon^2)$, $O(\varepsilon^2)$ and $O(\varepsilon^3)$. In any case the last term can be neglected if a second-order theory is retained.

In a similar way, the relationship of normal vectors of the body's wetted surface defined in the two frames of reference,

$$\vec{n}(x',y',z',t) = (n_1,n_2,n_3)$$

and

$$\vec{N}(x,y,z,t) = (N_1,N_2,N_3),$$

is

$$\vec{N} = \vec{n} + \mathbf{R}\vec{n} + \mathbf{H}\vec{n}, \tag{7}$$

2.2. Decomposition of the velocity potential and the pressure

Based on the assumptions of the fluid field, the velocity potential (unsteady velocity potential) around the moored floating body in the equilibrium frame may be decomposed into the form (Wu, 1984)

$$\phi(x,y,z,t) = \phi_I + \phi_D + \sum_{r=1}^{m} \phi_r, \tag{8}$$

where $\phi_I(x,y,z,t)$, $\phi_D(x,y,z,t)$ and $\phi_r(x,y,z,t)$ denote the incident wave potential, diffraction wave potential, and radiation wave potential arising from the responses of the flexible body. In frequency domain, the first-order unsteady velocity potential and the principal coordinates may be further expressed as

$$\phi = \mathrm{Re}\{[\varphi_I + \varphi_D + \sum_{r=1}^{m} \varphi_r p_r]e^{i\omega t}\}, \tag{9}$$

$$p_r(t) = \mathrm{Re}\{p_r e^{i\omega t}\}, \tag{10}$$

where ω is the wave circular frequency; $\varphi_I(x,y,z,\omega)$ and $\varphi_D(x,y,z,\omega)$ are components of the incident wave velocity potential and the diffraction wave potential respectively; $\varphi_r(x,y,z,\omega)$ $(r = 1,\ldots,m)$ is the components of the radiation wave potential arising from the vibration in the r-th principal dry mode of the flexible body, with unit amplitude and frequency ω; p_r is the complex amplitude of the principal coordinate. The sign Re{ } in Eqs. (9) and (10) denotes the real part of the complex in { }. The sign Re{ } is omitted in the following expressions of the potentials and the principal coordinates for clarity.

The governing equations of the fluid are the Laplace equation and the Lagrange integral equation. In equilibrium coordinate system, they may be expressed as

$$\nabla^2\phi(x,y,z,t) = 0, \tag{11}$$

$$\frac{p(x,y,z,t)}{\rho} + gz + \frac{v^2}{2} + \frac{\partial\phi}{\partial t} = 0. \tag{12}$$

$\vec{v}\,(x,y,z,t)$, $p(x,y,z,t)$ and ρ in Eq. (11) are respectively the velocity vector, pressure and density of the fluid, g is the gravity acceleration.

The fluid pressure acting on the mean wetted surface $\bar{S}$ during the motion and distortion of the body is given by the Bernoulli equation in the equilibrium axes system,

$$p(x,y,z,t)|_{\bar{S}} = -\rho\left[\frac{\partial\phi}{\partial t} + \frac{(\nabla\phi)^2}{2} + gz\right]_{\bar{S}}. \tag{13}$$

The movement of the body wetted surface from the steady state mean position S to the instantaneous position $S(t)$ induces the variation of the pressure. By employing Taylor's expansion from S to $S(t)$, the pressure may be expressed in the form

$$p|_{S(t)} = \left[1 + \vec{u}\cdot\nabla + \frac{(\vec{u}\cdot\nabla)(\vec{u}\cdot\nabla)}{2} + \ldots\right]p|_{\bar{S}}. \tag{14}$$

Substituting Eq. (13) into Eq. (14) and retaining the second-order $O(\varepsilon^2)$ in the square brackets, the pressure acting on the instantaneous wetted surface is

$$p|_{S(t)} = -\rho(1 + \vec{u}\cdot\nabla)\left[\frac{\partial\phi}{\partial t} + \frac{(\nabla\phi)^2}{2} + gz\right]_{\bar{S}}. \tag{15}$$

Considering Eq. (6) and substituting Eq. (5) into Eq. (15), one obtains

$$\begin{aligned} p|_{S(t)} = -\rho\Bigg\{&\left[\frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2 + gz\right] + (\vec{u}^{(1)}\cdot\nabla)\left[\frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2 + gz\right] \\ &+ (H\vec{r}\,'\cdot\nabla)\left[\frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2 + gz\right] + (R\vec{u}_D\cdot\nabla)\left[\frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2 + gz\right] \\ &+ (H\vec{u}_D\cdot\nabla)\left[\frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2 + gz\right]\Bigg\}_{\bar{S}}. \end{aligned} \tag{16}$$

Thus, the pressure acting on a three-dimensional flexible body with no special restrictions on its geometries can be represented in the form

$$p|_{S(t)} = p^{(1)}|_{\bar{S}} + p^{(2)}|_{\bar{S}}, \tag{17}$$

where

$$p^{(1)} = -\rho\left\{\frac{\partial\phi}{\partial t} + gz' + gw\right\}_{\bar{S}}, \tag{18}$$

$$p^{(2)} = -\rho\left\{g(\tilde{H}\vec{r}\,' + \tilde{R}\vec{u}_D)\cdot\nabla z' + (\vec{u}^{(1)}\cdot\nabla)\frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2\right\}_{\bar{S}}. \tag{19}$$

If the second order forces due to the influence of the rigid body rotation are to be considered, the second-order generalized forces can be expressed as

$$Z_r(t) = -\int\int_{S(t)} \vec{N}\cdot\vec{u}_r^{\,0} p\,\mathrm{d}\,S. \tag{20}$$

In doing so, it is implicitly assumed that the structure is linear, and in the consequential analysis of non-linear response

$$\vec{u}_r^{\,0}\ (r = 1,2,\ldots,m)$$

is used as a set of orthogonal functions although the rigid body modes are not exactly orthogonal. This has been practiced in the linear theory of hydeoelasticity (Wu, 1984; Price and Wu, 1985; Bishop et al., 1986).

Substitution of Eqs. (17–19) and (7) into Eq. (20) yields

$$Z_r(t) = Z_r^{(0)} + Z_r^{(1)}(t) + Z_r^{(2)}(t), \tag{21}$$

where $Z_r^{(0)}$, $Z_r^{(1)}(t)$ and $Z_r^{(2)}(t)$ are the generalized constant, first- and second-order forces respectively.

The constant forces are

$$Z_r^{(0)} = \rho \iint_{\bar{S}} \vec{n} \cdot \vec{u}_r^{\,0} g z' \mathrm{d}\, S. \tag{22}$$

These provide the generalized steady state buoyancy forces.

The generalized first-order forces may be written as

$$Z_r^{(1)}(t) = E_r^{(1)}(t) + H_r^{(1)}(t) + R_r^{(1)}(t) + \Delta R_r(t), \tag{23}$$

with

$$E_r^{(1)}(t) = \rho \iint_{\bar{S}} \vec{n} \cdot \vec{u}_r^{\,0} \frac{\partial}{\partial t}[\phi_I(t) + \phi_D(t)] \mathrm{d}\, S, \tag{24}$$

$$H_r^{(1)}(t) = \sum_{k=1}^{m} \rho \iint_{\bar{S}} \vec{n} \cdot \vec{u}_r^{\,0} \frac{\partial}{\partial t} \phi_k(t) \mathrm{d}\, S, \tag{25}$$

$$R_r^{(1)}(t) = \rho \iint_{\bar{S}} \vec{n} \cdot \vec{u}_r^{\,0} g w \mathrm{d}\, S, \tag{26}$$

$$\Delta R_r(t) = \rho \iint_{\bar{S}} (\mathbf{R}\vec{n}) \cdot \vec{u}_r^{\,0} g z' \mathrm{d}\, S, \tag{27}$$

where $E_r^{(1)}(t)$, $H_r^{(1)}(t)$ and $R_r^{(1)}(t)$ are the generalized first-order wave exciting forces, radiation forces and restoring forces. They are exactly the same as those in the linear three-dimensional hydroelasticity theory (Wu, 1984; Price and Wu, 1985; Bishop et al., 1986; Du, 1996; Wang, 1996). $\Delta R_r(t)$ denotes the influence from the rotation of the rigid body modes to the first order forces, which does not include in the linear hydroelasticity theory.

The second order forces may be represented in the form

$$Z_r^{(2)}(t) = F_r^{(2)}(t) + E_r^{(2)}(t) + D_r^{(2)}(t) + S_r^{(2)}(t) + \Delta Z_r^{(2)}(t), \tag{28}$$

with

$$F_r^{(2)}(t) = \rho \iint_{\bar{S}} [(\mathbf{R}\vec{n}) \cdot \vec{u}_r^{\,0} + (\vec{n} \cdot \vec{u}_r^{\,0})(\vec{u}^{\,(1)} \cdot \nabla)] \frac{\partial}{\partial t}[\phi(t)_I + \phi_D(t)] \mathrm{d}S \tag{29}$$
$$+ \rho \iint_{\bar{S}} \vec{n} \cdot \vec{u}_r^{\,0} \frac{1}{2}[\nabla \phi_I(t) + \nabla \phi_D(t)]^2 \mathrm{d}S,$$

$$E_r^{(2)}(t) = \sum_{k=1}^{m} \rho \iint_{\bar{S}} \vec{n} \cdot \vec{u}_r^{\,0} \nabla[\phi_I(t) + \phi_D(t)] \nabla \phi_k(t) \mathrm{d}S, \tag{30}$$

$$D_r^{(2)}(t) = \sum_{k=1}^{m} \rho \iint_{\bar{S}} [(\mathbf{R}\vec{n})\cdot\vec{u}_r^{\,0} + (\vec{n}\cdot\vec{u}_r^{\,0})(\vec{u}^{\,(1)}\cdot\nabla)]\frac{\partial}{\partial t}\phi_k(t)\mathrm{d}S \tag{31}$$

$$+ \sum_{k=1}^{m}\sum_{l=1}^{m} \frac{1}{2}\rho \iint_{\bar{S}} (\vec{n}\cdot\vec{u}_r^{\,0})\nabla\phi_k(t)\cdot\nabla\phi_l(t)\mathrm{d}S,$$

$$S_r^{(2)}(t) = \rho \iint_{\bar{S}} (\mathbf{R}\vec{n})\cdot\vec{u}_r^{\,0} g(w + z')\mathrm{d}S + \rho \iint_{\bar{S}} (\vec{n}\cdot\vec{u}_r^{\,0})(\mathbf{H}\vec{r}\,' \tag{32}$$

$$+ \mathbf{R}\vec{u}_d)\cdot\nabla(gz')\mathrm{d}S,$$

$$\Delta Z_r^2(t) = \rho \int\int_{\Delta S} \vec{n}\cdot\vec{u}_r^{\,0}\left[\frac{\partial\phi}{\partial t} + g(z' + w)\right]\mathrm{d}S. \tag{33}$$

Here $F_r^{(2)}(t)$ contains all the terms related to the incident and diffracted wave potentials, and is referred to the generalized second-order wave exciting force. $E_r^{(2)}(t)$ denotes the crossing terms containing the products of the radiation wave velocity and the incident-diffracted wave velocities. $D_r^{(2)}(t)$ represents all the terms related to the radiation potentials. We define them as the generalized second-order radiation force. $S_r^{(2)}(t)$ collects all the second order steady terms. $\Delta Z_r^{(2)}(t)$ describe the forces induced by the instantaneous variation of the wetted surface of the structure. $\Delta S = S - \bar{S}$ is the difference of the instantaneous wetted surface $S(t)$ and the mean wetted surface $\bar{S}$.

2.3. The generalized forces in irregular waves

2.3.1. Irregular waves

According to Pierson (1955), the irregular waves may be described by the aggregation of regular waves defined at the origin of the equilibrium frame. That is

$$\zeta(t) = \sum_{j=1}^{N} \zeta_j \cos(\omega_j t + \varepsilon_j), \tag{34}$$

where N is the number of the regular wave components. $\omega_j,\kappa_j,\varepsilon_j$, and ζ_j are respectively the wave circular frequency, the wave number, the random phase angle and the wave amplitude of the j-th components. The wave amplitude may be obtained from the wave energy spectrum.

Using Eq. (34), the first-order incident wave potential, the diffracted wave potential and the principal coordinates may be expressed as

$$\phi_I(x,y,z,t) = \sum_{j=1}^{N} \zeta_j \varphi_I(\omega_j) e^{i(\omega_j t+\varepsilon_j)}, \tag{35}$$

$$\phi_D(x,y,z,t) = \sum_{j=1}^{N} \zeta_j \varphi_D(\omega_j) e^{i(\omega_j t+\varepsilon_j)}, \tag{36}$$

$$p_k(t) = \sum_{j=1}^{N} \zeta_j p_k(\omega_j) e^{i(\omega_j t+\varepsilon_j)}. \tag{37}$$

The sign Re{ } is omitted from above three equations just as mentioned in section 2.2.

2.3.2. *The generalized first-order forces*

Substituting Eqs. (35) and (36) into Eq. (24), one obtains the generalized first-order wave exciting forces

$$E_r^{(1)}(t) = \sum_{j=1}^{N} \zeta_j \xi_r(\omega_j) e^{i(\omega_j t+\varepsilon_j)}, \tag{38}$$

where $\xi_r(\omega_j)$, the coefficients of the generalized first-order wave exciting forces, can be expressed as

$$\xi_r(\omega_j) = \rho \iint_{\bar{S}} \vec{n} \cdot \vec{u}_r^{\,0} (\mathrm{i}\omega_j)[\varphi_I(\omega_j) + \varphi_D(\omega_j)]\mathrm{d}S. \tag{39}$$

It is well known that the motion of a linear structure can be described as the aggregation of the motions of all the dry modes, that is

$$\vec{u}^{\,(1)} = \sum_{r=1}^{m} \vec{u}_r(x',y',z',t) = \sum_{r=1}^{m} \vec{u}_r^{\,0} p_r(t). \tag{40}$$

where $\vec{u}_r^{\,0}(x',y',z')=(u_r^0,v_r^0,w_r^0)$ $(r = 1,2,\ldots,m)$ is a set of the principal modes of the structure in vacuum. The k-th radiation potential may be accordingly expressed as

$$\phi_k(x,y,z,t) = \varphi_k(x,y,z)p_k(t). \tag{41}$$

Substituting Eqs. (37) and (41) into Eq. (25), one obtains the first-order radiation forces

$$H_r^{(1)}(t) = \sum_{k=1}^{m} \sum_{j=1}^{N} \zeta_j [\omega_j^2 A_{rk}(\omega_j) - i\omega_{ej} B_{rk}(\omega_j)] p_k(\omega_j) e^{i(\omega_j t+\varepsilon_j)}, \tag{42}$$

where $A_{rk}(\omega_j)$ and $B_{rk}(\omega_j)$ are coefficients of added mass and added damping defined as

$$\begin{matrix} A_{rk}(\omega) = \dfrac{1}{\omega^2}\mathrm{Re} \\ B_{rk}(\omega) = \dfrac{i}{\omega}\mathrm{Im} \end{matrix} \{\mathrm{i}\rho \iint_{\bar{S}} \vec{n} \cdot \vec{u}_r^{\,0} \omega \varphi_k(\omega)\mathrm{d}S\}. \tag{43}$$

Substituting Eqs. (37) and (40) into Eq. (26), one obtains the generalized first-order restoring forces

$$R_r^{(1)}(t) = \sum_{k=1}^{m} C_{rk} \sum_{j=1}^{N} \zeta_j p_k(\omega) e^{i(\omega_j t + \varepsilon_j)}, \tag{44}$$

where C_{rk} are the frequency independent coefficients of the generalized first-order forces, and are expressed as

$$C_{rk} = \rho \iint_{\bar{S}} \vec{n} \cdot \vec{u}_r^{\,0} g w_k^0 \mathrm{d}S. \tag{45}$$

By employing Eq. (27) and denoting

$$(\mathbf{R}\vec{n}) \cdot \vec{u}_r^{\,0} = \sum_{k=4}^{6} d_{rk} p_k(t), \tag{46}$$

with

$$\begin{cases} d_{r4} = w_r^0 n_2 - v_r^0 n_3, & d_{r5} u_r^0 n_3 - w_r^0 n_1, \\ d_{r6} = v_r^0 n_1 - u_r^0 n_2, & d_{rk} = 0, \end{cases} (k \neq 4,5,6). \tag{47}$$

The generalized first-order restoring forces described in Eq. (27) may be expressed as

$$\Delta R_r(t) = \sum_{k=1}^{m} \Delta C_{rk} \sum_{j=1}^{N} \zeta_j p_k(\omega_j) e^{i(\omega_j t + \varepsilon_j)}, \tag{48}$$

where ΔC_{rk} are the first order restoring coefficients

$$\Delta C_{rk} = \rho \iint_{\bar{S}} d_{rk} g z' \mathrm{d}S. \tag{49}$$

2.3.3. The generalized second-order forces

It has been customarily implied in the previous sections that real values are taken when the time variations and the corresponding phase differences for the potentials and the principal coordinates are represented in complex forms. When this convention is used to the case of the second-order forces, the following relation is to be followed.

$$\mathrm{Re}(X) \cdot \mathrm{Re}(Y) = \frac{1}{2}[\mathrm{Re}(X \cdot \bar{Y}) + \mathrm{Re}(X \cdot Y)], \tag{50}$$

where X and Y are two arbitrary chosen complex functions or quantities. Here and hereafter an over bar on a complex function or quantity represents its conjugation.

Using Eq. (50) and substituting Eqs. (46) and (40) into Eqs. (35–37), and then further substituting them into Eq. (29), one obtains the generalized second-order forces

$$F_r^{(2)}(t) = \sum_{k=1}^{m}\sum_{i=1}^{N}\sum_{j=1}^{N}\zeta_i\zeta_j\xi_{rk}(\omega_i)\frac{1}{2}\{\bar{p}_k(\omega_j)e^{i[(\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)]} + p_k(\omega_j)e^{i[(\omega_i+\omega_j)t+(\varepsilon_i+\varepsilon_j)]}\} + \sum_{i=1}^{N}\sum_{j=1}^{N}\zeta_i\zeta_j\frac{1}{2}\{f_r^*(\omega_i,\omega_j)e^{i[(\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)]} + f_r(\omega_i,\omega_j)e^{i[(\omega_i+\omega_j)t+(\varepsilon_i+\varepsilon_j)]}\}, \tag{51}$$

where

$$\xi_{rk}(\omega_i) = i\omega_i\rho\iint_{\bar{S}}[d_{rk} + (\vec{n}\cdot\vec{u}_r^{\,0})(\vec{u}_k^{\,0}\cdot\nabla)][\varphi_I(\omega_i) + \varphi_D(\omega_i)]\mathrm{d}S, \tag{52}$$

$$f_r(\omega_i,\omega_j) = \rho\iint_{\bar{S}}(\vec{n}\cdot\vec{u}_r^{\,0})\frac{1}{2}\nabla[\varphi_I(\omega_i) + \varphi_D(\omega_i)]\nabla[\varphi_I(\omega_j) + \varphi_D(\omega_j)]\mathrm{d}S, \tag{53}$$

$$f_r^*(\omega_i,\omega_j) = \rho\iint_{\bar{S}}(\vec{n}\cdot\vec{u}_r^{\,0})\frac{1}{2}\nabla[\varphi_I(\omega_i) + \varphi_D(\omega_i)]\nabla[\bar{\varphi}_I(\omega_j) + \bar{\varphi}_D(\omega_j)]\mathrm{d}S. \tag{54}$$

Using Eq. (50) and substituting Eqs. (41) and (35–37) into Eq. (30), one obtains

$$E_r^{(2)}(t) = \sum_{k=1}^{m}\sum_{i=1}^{N}\sum_{j=1}^{N}\zeta_i\zeta_j p_k(\omega_i)\frac{1}{2}\{h_{rk}^*(\omega_i,\omega_j)e^{i[(\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)]} + h_{rk}(\omega_i,\omega_j)e^{i[(\omega_i+\omega_j)t+(\varepsilon_i+\varepsilon_j)]}\}, \tag{55}$$

where

$$h_{rk}(\omega_i,\omega_j) = \rho\iint_{\bar{S}}(\vec{n}\cdot\vec{u}_r^{\,0})\nabla[\varphi_I(\omega_j) + \varphi_D(\omega_j)]\cdot\nabla\varphi_k(\omega_i)\mathrm{d}S \tag{56}$$

$$h_{rk}^*(\omega_i,\omega_j) = \rho\iint_{\bar{S}}(\vec{n}\cdot\vec{u}_r^{\,0})\nabla[\bar{\varphi}_I(\omega_j) + \bar{\varphi}_D(\omega_j)]\cdot\nabla\varphi_k(\omega_i)\mathrm{d}S. \tag{57}$$

Using Eq. (50) and substituting Eqs. (46), (40), (41) and (37) into Eq. (31), one obtains the generalized second-order forces

$$D_r^{(2)}(t) = \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{N}\sum_{j=1}^{N}\zeta_i\zeta_j p_k(\omega_i)\frac{1}{2}\{\bar{p}_l(\omega_j)[q_{rkl}(\omega_i,\omega_j) + t_{rkl}^*(\omega_i,\omega_j)]\cdot e^{i[(\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)]} + p_l(\omega_j)[q_{rkl}(\omega_i,\omega_j) \tag{58}$$

$$+ t_{rkl}(\omega_i,\omega_j)]\cdot e^{i[(\omega_i+\omega_j)t+(\varepsilon_i+\varepsilon_j)]}\},$$

where the coefficients of the generalized second-order forces are

$$q_{rkl}(\omega_i) = i\omega_i\rho\iint_{\bar{S}}[d_{rk} + (\vec{n}\cdot\vec{u}_r^{\,0})(\vec{u}_l^{\,0}\cdot\nabla)]\varphi_k(\omega_i)\mathrm{d}S, \tag{59}$$

$$t^*_{rkl}(\omega_i,\omega_j) = \rho\iint_{\bar{S}}(\vec{n}\cdot\vec{u}_r^{\,0})\frac{1}{2}\nabla\varphi_k(\omega_i)\nabla\varphi_l(\omega_j)\mathrm{d}S, \tag{60}$$

$$t_{rkl}(\omega_i,\omega_j) = \rho\iint_{\bar{S}}(\vec{n}\cdot\vec{u}_r^{\,0})\frac{1}{2}\nabla\varphi_k(\omega_i)\nabla\bar{\varphi}_l(\omega_j)\mathrm{d}S. \tag{61}$$

Similar to Eq. (46), one has

$$(\mathbf{H}\vec{n})\cdot\vec{u}_r^{\,0} = \sum_{k=1}^{m}\sum_{l=1}^{m}\alpha_{rkl}p_l(t)p_k(t). \tag{62}$$

The nonzero elements of α_{rkl} are

$$\begin{cases}\alpha_{r44} = -\frac{1}{2}(v_r^0n_2 + w_r^0n_3), & \alpha_{r55} = -\frac{1}{2}(u_r^0n_1 + w_r^0n_3),\\ \alpha_{r45} = \alpha_{r54} = \frac{1}{2}v_r^0n_1, & \alpha_{r46} = \alpha_{r64} = \frac{1}{2}w_r^0n_1,\\ \alpha_{r56} = \alpha_{r65} = \frac{1}{2}w_r^0n_2, & \alpha_{r66} = -\frac{1}{2}(u_r^0n_1 + v_r^0n_2).\end{cases} \tag{63}$$

From Eqs. (1) and (40), one can easily find

$$\vec{u}_D = \sum_{r=7}^{m}\{u_r,\ v_r,\ w_r\} = \sum_{r=7}^{m}\vec{u}_r^{\,0}p_r(t). \tag{64}$$

By employing Eqs. (46) and (50) and introducing the expression

$$(\mathbf{H}\vec{r}\,' + \mathbf{R}\vec{u}_D)\cdot\nabla(gz') = \sum_{k=1}^{m}\sum_{l=1}^{m}e_{kl}p_l(t)p_k(t), \tag{65}$$

with

$$\begin{cases} e_{44} = e_{55} = -\frac{1}{2}gz', \; e_{45} = e_{54} = e_{66} = 0, \\ e_{46} = e_{64} = \frac{1}{2}gx', \; e_{56} = e_{65} = \frac{1}{2}gy', \\ e_{4l} = \frac{1}{2}gv_l^0, \; e_{5l} = -\frac{1}{2}gu_l^0, \; (l \geq 7), e_{kl} = 0, \; (\text{other } k,l) \end{cases} . \tag{66}$$

Eq. (32) may be rewritten as

$$S_r^{(2)}(t) = \sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{N}\sum_{j=1}^{N}\zeta_i\zeta_j g_{rkl}\frac{1}{2}p_k(\omega_i)\{\bar{p}_l(\omega_j)e^{i[(\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)]} + p_l(\omega_j)e^{i[(\omega_i+\omega_j)t+(\varepsilon_i+\varepsilon_j)]}\}, \tag{67}$$

where

$$g_{rkl} = \rho\iint_{\bar{S}} d_{rk}gw_k^0 \mathrm{d}S + \rho\iint_{\bar{S}} \alpha_{rkl}gz'\mathrm{d}S + \rho\iint_{\bar{S}} (\overrightarrow{n}\cdot\overrightarrow{u}_r^0)e_{kl}\cdot\mathrm{d}S. \tag{68}$$

With the surface integral on ΔS being converted to the line integral along the waterline C_W, and the wave elevation on the waterline being expressed as

$$\zeta^* = -\frac{1}{g}\frac{\partial\phi}{\partial t}\bigg|_{C_W on \bar{S}}. \tag{69}$$

Eq. (33), the generalized second-order forces due to the variation of instantaneous wetted surface, may be represented as

$$\Delta Z_r^2(t) = \rho g\int_{C_W}\frac{(\overrightarrow{n}\cdot\overrightarrow{u}_r^0)}{\sqrt{1-n_3^2}}\mathrm{d}l\int_0^{\zeta^*-w}[z'-\zeta^* + w]\mathrm{d}z' = -\frac{1}{2}\rho g\int_{C_W}\tilde{z}^2(\overrightarrow{n}\cdot\overrightarrow{u}_r^0)\frac{\mathrm{d}l}{\sqrt{1-n_3^2}}, \tag{70}$$

where $\tilde{z}$ denotes the relative vertical displacement of the body and the wave surface:

$$\tilde{z} = \zeta^* - w. \tag{71}$$

Using Eq. (50) and substituting Eqs. (35–37), (41), (7), (40), (69) and (71) into Eq. (70), one obtains

$$\Delta Z_r^{(2)}(t) = J_{r0} + \sum_{j=1}^{N}\zeta_j[\bar{J}_r(\omega_j)e^{-i(\omega_j t+\varepsilon_j)} + 3J_r(\omega_j)e^{i(\omega_j t+\varepsilon_j)}]$$

$$+\sum_{k=1}^{m}\sum_{j=1}^{N}\zeta_j[\bar{J}_{rk}(\omega_j)\bar{p}_k(\omega_j)e^{-i(\omega_j t+\varepsilon_j)} + 3J_{rk}(\omega_j)p_k(\omega_j)e^{i(\omega_j t+\varepsilon_j)}]$$

$$+\sum_{i=1}^{N}\sum_{j=1}^{N}\zeta_i\zeta_j\{K_r^*(\omega_i,\omega_j)e^{i[(\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)]} + K_r(\omega_i,\omega_j)e^{i[(\omega_i+\omega_j)t+(\varepsilon_i+\varepsilon_j)]}\}$$

$$+\sum_{k=1}^{m}\sum_{i=1}^{N}\sum_{j=1}^{N}\zeta_i\zeta_j\{[\bar{K}_{rk}(\omega_i,\omega_j)\bar{p}_k(\omega_j) + K_{rk}(\omega_i,\omega_j)p_k(\omega_j)]\cdot e^{i[(\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)]} \quad (75)$$

$$+ K_{rk}^*(\omega_i,\omega_j)p_k(\omega_j)e^{i[(\omega_i+\omega_j)t+(\varepsilon_i+\varepsilon_j)]}\}$$

$$+\sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{N}\sum_{j=1}^{N}\zeta_i\zeta_j\{G_{rkl}^*(\omega_i,\omega_j)p_k(\omega_i)\bar{p}_l(\omega_j)e^{i[(\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)]}$$

$$+ G_{rkl}(\omega_i,\omega_j)p_k(\omega_i)p_l(\omega_j)e^{i[(\omega_i+\omega_j)t+(\varepsilon_i+\varepsilon_j)]}\}.$$

The coefficients contained in these formulas are given in Appendix A.

It is not difficult to calculate the second-order forces and their coefficients under the condition that the first-order incident wave potential, diffracted wave potential, radiated wave potential and the first order principal coordinates have all been obtained.

3. Coefficients of mooring forces

It is assumed that there are J mooring lines, and Cm_{rk}^j are elements of the restoring force matrix of the j-th mooring line. They may be expressed as

$$Cm_{rk}^j = [\overrightarrow{u}_{rj}^{\,0}]^T S \overrightarrow{u}_{kj}^{\,0}, \tag{76}$$

with

$$\mathbf{S} = \frac{E's}{L}\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}, \tag{77}$$

and

$$\begin{cases} S_{11} = \cos^2\alpha\cos^2\beta,\ S_{12} = S_{21} = \cos^2\alpha\cos\beta\sin\beta, \\ S_{13} = S_{31} = -\cos\alpha c\sin\alpha\cos\beta,\ S_{22} = \cos^2\alpha\sin^2\beta, \\ S_{23} = S_{32} = -\cos\alpha\sin\alpha\sin\beta,\ S_{33} = \sin\alpha, \end{cases} \tag{78}$$

where α is the angle between the horizontal and the tangent directions of the mooring line at the mooring point on the floating body. β is the angle between the horizontal projection of the mooring line and the x direction. E', s and L are Young's module, the section area and the length of the mooring line respectively.

The $\overrightarrow{u}_{rj}^{0}$ in Eq. (76) are displacement modes (Chen, 2001) of the mooring point on the floating body of the j-th mooring line. They may be expressed as

$$\overrightarrow{u}_{rj}^{0} = \mathbf{l}^{-1}\mathbf{NL}U_{ej}, \tag{79}$$

U_{ej} are the modes of the structural element where the mooring point of the j-th mooring line belongs to. $\mathbf{l}$ is the transformation matrix between the local coordinate system of the structural element and the whole coordinate system of the floating body. $\mathbf{L}$ is a unit matrix. $\mathbf{N}$ is the embedding matrix of the structural element.

4. Equations of motion

Based on the solutions of the generalized fluid forces and the mooring restoring forces acting on the flexible floating body, the equations of motion for solving the principal coordinates $p_k(t)$, with both the first-order and second-order actions are included, may be represented as

$$\begin{aligned}\sum_{k=1}^{m} &[(a_{rk} + A_{rk})\ddot{p}_k(t) + (b_{rk} + B_{rk})\dot{p}_k(t) + (c_{rk} + C_{rk} + \Delta C_{rk} \\ &+ Cm_{rk})p_k(t)] = E_r^{(1)}(t) + Z_r^{(2)}(t) + Z_r^{(0)} + Q_r = Z_r^{(0)} + Q_r \\ &+ \sum_{i=0}^{N}\sum_{j=1}^{N}\zeta_i\zeta_j Q_{rij}e^{i[(\omega_i-\omega_j)t+(\varepsilon_i-\varepsilon_j)]} + \sum_{i=0}^{N}\sum_{j=1}^{N}\zeta_i\zeta_j D_{rij}e^{i[(\omega_i+\omega_j)t+(\varepsilon_i+\varepsilon_j)]},\end{aligned} \tag{80}$$

where a_{rk}, b_{rk} and c_{rk} are the elements of the generalized mass matrix, the generalized damping matrix and the generalized rigid matrix of the structure respectively; Q_r are the generalized gravity forces. The difference frequency forces and the sum frequency forces in Eq. (80) may be expressed as

$$\begin{aligned}Q_{rij} = {}& K_r^*(\omega_i,\omega_j) + \frac{1}{2}f_r^*(\omega_i,\omega_j) + \frac{1}{2}\sum_{k=1}^{m}\{\xi_{rk}(\omega_i)\bar{p}_k^{(1)}(\omega_j) \\ &+ h_{rk}^*(\omega_i,\omega_j)p_k^{(1)}(\omega_i) + 2\bar{K}_{rk}(\omega_i,\omega_j)\bar{p}_k^{(1)}(\omega_j) + 2K_{rk}(\omega_i,\omega_j)p_k^{(1)}(\omega_i) \\ &+ \frac{1}{2}p_k^{(1)}(\omega_i)\sum_{l=1}^{m}[\bar{p}_l^{(1)}(\omega_j)q_{rkl}(\omega_i,\omega_j) + \bar{p}_l^{(1)}(\omega_j)t_{rkl}^*(\omega_i,\omega_j) + g_{rkl}\bar{p}_l^{(1)}(\omega_j) \\ &+ 2G_{rkl}^*(\omega_i,\omega_j)\bar{p}_l^{(1)}(\omega_j)]\},\end{aligned} \tag{81}$$

$$Q_{r0j} = 0, \tag{82}$$

$$D_{r0j} = \rho \iint_{\bar{S}} \overrightarrow{n} \cdot \overrightarrow{u}\,{}^{0}_{r}(i\omega_j)[\varphi_I(\omega_j) + \varphi_D(\omega_j)]\mathrm{d}S, \tag{83}$$

$$\begin{aligned} D_{rij} = {} & K_r(\omega_i,\omega_j) + \frac{1}{2}f_r(\omega_i,\omega_j) + \frac{1}{2}\sum_{k=1}^{m} \{\xi_{rk}(\omega_i)p_k^{(1)}(\omega_j) \\ & + h_{rk}(\omega_i,\omega_j)p_k^{(1)}(\omega_i) + 2K^*_{rk}(\omega_i,\omega_j)p_k^{(1)}(\omega_j) \\ & + \frac{1}{2}p_k^{(1)}(\omega_i)\sum_{l=1}^{m} [p_l^{(1)}(\omega_j)q_{rkl}(\omega_i,\omega_j) + p_l^{(1)}(\omega_j)t_{rkl}(\omega_i,\omega_j) + g_{rkl}p_l^{(1)}(\omega_j) \\ & + 2G_{rkl}(\omega_i,\omega_j)p_l^{(1)}(\omega_j)]\}. \end{aligned} \tag{84}$$

Equation (80) shows that the solution of the total responses $p_k(t)$ consists of the steady components due to the excitation of $Z_r^{(0)}$ and Q_r, the wave frequency components due to the excitation from the terms of Q_{r0j}andD_{r0j}, the difference frequency components coming from the excitation terms of Q_{rij} and the sum frequency components coming from those of D_{rij}. It is then proper to represent the solution in the form

$$p_k(t) = \bar{p}_k + \sum_{i=0}^{N}\sum_{j=1}^{N} \zeta_i\zeta_j\{p_k^-(\omega_{ij}^-)e^{i[\omega_{ij}^- t+\varepsilon_{ij}^-]} + p_k^+(\omega_{ij}^+)e^{i[\omega_{ij}^+ t+\varepsilon_{ij}^+]}\}, \tag{85}$$

where

$$\omega_{ij}^- = \omega_i - \omega_j,\ \omega_{ij}^+ = \omega_i + \omega_j,\ \varepsilon_{ij}^- = \varepsilon_i - \varepsilon_j,\ \varepsilon_{ij}^+ = \varepsilon_i + \varepsilon_j. \tag{86}$$

Obviously, $\bar{p}_k$ are steady components. $p_k^-(\omega_{0j}^-)$ and $p_k^+(\omega_{0j}^+)$=$p_k^{(1)}(\omega_j)$ are wave frequency components. $p_k^-(\omega_{ij}^-)$ and $p_k^+(\omega_{ij}^+)$ defined are unsteady difference frequency and sum frequency principal coordinates respectively. The equations for solving these components are as follows.

$$\sum_{k=1}^{m} (c_{rk} + C_{rk} + \Delta C_{rk} + Cm_{rk})\bar{p}_k = Z_r^{(0)} + Q_r,\ (r = 1,\ldots,m), \tag{87}$$

$$\begin{aligned} & \sum_{k=1}^{m} [-(\omega_{ij}^-)^2(a_{rk} + A_{rk}) + (i\omega_{ij}^-)(b_{rk} + B_{rk}) + (c_{rk} + C_{rk} + \Delta C_{rk} \\ & + Cm_{rk})]p_k^-(\omega_{ij}^-) = Q_{rij},(r = 1,2,\ldots,m)\ (i,j-1 = 0,1\ldots,N), \end{aligned} \tag{88}$$

$$\begin{aligned} & \sum_{k=1}^{m} [-(\omega_{ij}^+)^2(a_{rk} + A_{rk}) + (i\omega_{ij}^+)(b_{rk} + B_{rk}) + (c_{rk} + C_{rk} + \Delta C_{rk} \\ & + Cm_{rk})]p_k^+(\omega_{ij}^+) = D_{rij},\ (r = 1,2,\ldots,m)\ (i,j-1 = 0,1,\ldots,N). \end{aligned} \tag{89}$$

In the linear complex Eqs. (88) and (89), the first-order hydrodynamic coefficients A_{rk} and B_{rk} are calculated by employing Eq. (43) at the corresponding frequencies $\omega = |\omega_{ij}|$ or $\omega = \omega_{ij}$ for Eq. (43) respectively.

Similar to the response equation of the total principal coordinates, the response equations of the first-order principal coordinates, the difference frequency principal coordinates, and the sum frequency principal coordinates can be expressed as

$$p_k^{(1)}(t) = \sum_{j=1}^{N} \zeta_j p_k(\omega_{ej}) e^{i(\omega_{ej}t+\varepsilon_j)}, \tag{90}$$

$$p_k^-(t) = \sum_{i=1}^{N}\sum_{j=1}^{N} \zeta_i\zeta_j p_k^-(\omega_{ij}^-) e^{i[\omega_{ij}^- t+\varepsilon_{ij}^-]}, \tag{91}$$

$$p_k^+(t) = \sum_{i=1}^{N}\sum_{j=1}^{N} \zeta_i\zeta_j p_k^+(\omega_{ij}^+) e^{i[\omega_{ij}^+ t+\varepsilon_{ij}^+]}. \tag{92}$$

When the responses of the principal coordinates have been obtained, one can calculate the vertical displacement, the vertical bending moment and the vertical shearing force using the following equations:

$$\begin{cases} w(t) = \sum_{r=1}^{m} w_r p_r(t), \\ M(t) = \sum_{r=7}^{m} M_r p_r(t), \\ V(t) = \sum_{r=7}^{m} V_r p_r(t), \end{cases} \tag{93}$$

where w_r, M_r and V_r are dynamic displacement, bending moment and shearing force modes of the floating body (Wu, 1984; Du, 1996; Chen, 2001).

5. Numerical example

5.1. The model

A moored box-type barge is chosen to illustrate the theory presented above. The particulars of the barge and the natural frequencies as well as the mode characteristics of the barge and the mooring system are shown respectively in Table 1 and Table 2. The abbreviations VB and HB in Table 2 represent the vertical bending and horizontal bending respectively.

5.2. Discussion of the first-order principal coordinates

If only the actions of the first-order forces D_{r0j} are considered, Eq. (80) may be degenerated to linear hydroelasticity equations of a moored floating flexible body. By solving the linear equations, the generalized first-order principal coordinates of

Table 1
The particulars of the barge

Barge		Mooring system	
Length	120 m	Mooring line number	4
Width	14 m	Gravity of unit length	813.4 N/m
Height	8 m	Length of the line	70 m
Drift	5.749 m	Vertical and horizontal distances between the mooring points and pickets	22.43 m and 62.85 m
Plate thickness	0.01 m	Young's modulus	3600 MPa
Young's modulus	207000 MPa	Mooring angles	30°, 150°, 210°, 310°

Table 2
Natural frequencies and generalized mass coefficients for the first 10 flexible modes

Serial number	Vibration type	Natural frequency (rad/s)	Generalized mass	Mode characteristic
7	Two-node VB	5.762230	2399992.075961	Port-starboard symmetric
8	Two-node HB	8.794543	2399992.075961	Port-starboard antisymmetric
9	Three-node VB	15.883809	2399988.010947	Port-starboard symmetric
10	One-node torsion	17.325088	185405838.355888	Port-starboard antisymmetric
11	Three-node HB	24.242497	2399988.010947	Port-starboard antisymmetric
12	Axial tension	28.505832	4799987.003524	Port-starboard symmetric
13	Four-node VB	31.138616	2399982.130573	Port-starboard symmetric
14	Two-node torsion	34.650176	185405618.594298	Port-starboard antisymmetric
15	Four-node HB	47.524987	2399982.130573	Port-starboard antisymmetric
16	Five-node VB	51.473713	2399974.341913	Port-starboard symmetric

a moored floating body may be obtained. The numerical results of the first 11 modes (including six rigid modes) are shown in Fig. 2. The wave angle is β=135°.

Based on the curves in Fig. 2, the frequency characteristics of the generalized first-order principal coordinates of a moored floating body under the wave angle β=135° can be analyzed as follows.

1. The first-order principal coordinates approach zero when the frequencies approach the infinite. The first-order principal coordinate of the heave mode approaches 1.0 when the frequency approaches zero.
2. The first-order principal coordinate of the sway mode is higher than that of the surge mode. The first-order principal coordinate of the roll mode is higher than that of the pitch mode. The first-order principal coordinate of the two-nodes VB mode is higher than that of the three-nodes VB mode. The results which have not been shown in Fig. 2 indicate that the first-order principal coordinates of the lower modes are higher than that of the higher modes for the same modes type.

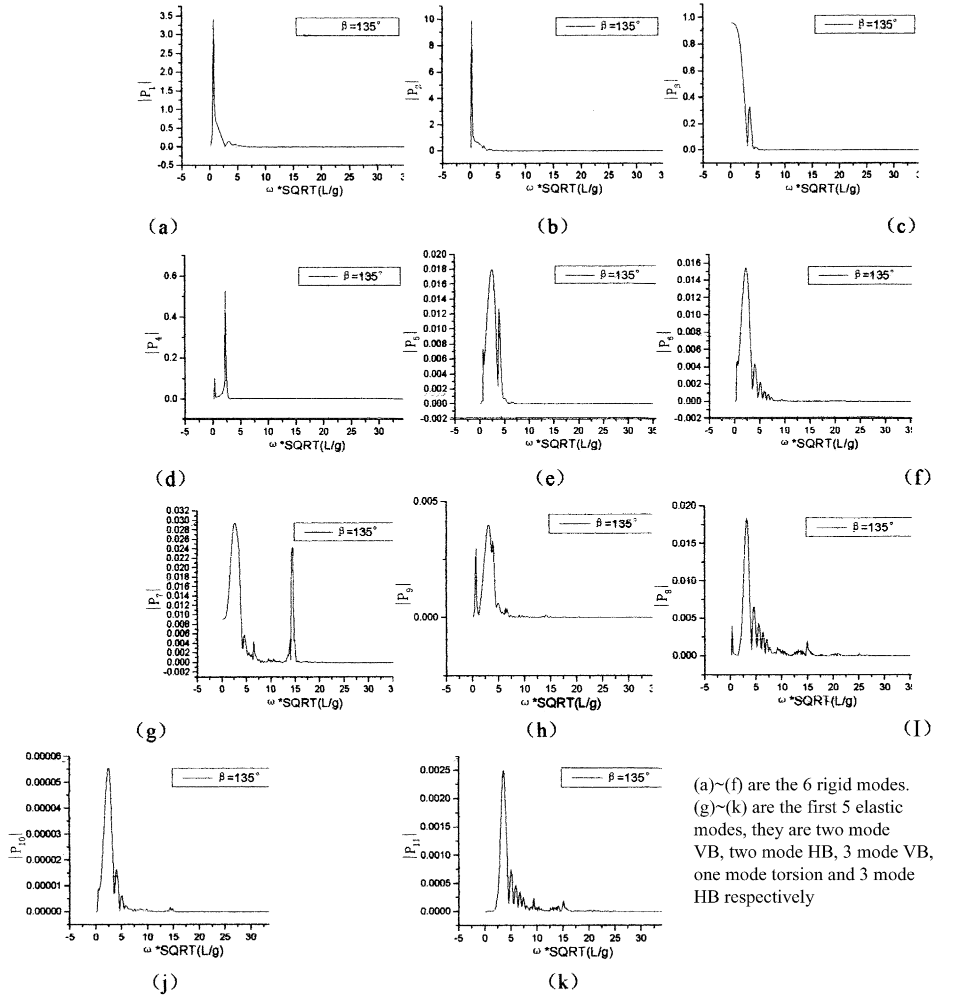


Fig. 2. The first order principal coordinate of the first 11 modes.

3. Peaks can be found on the curves, and all of them have their corresponding physical meanings. They correspond to two types of resonance. One is the resonance of the mode itself, and the other is the resonance coupled with the other modes. They can be analyzed as follows.

Each mode of the moored floating body presented in this paper has its corresponding wet resonant frequency ω_{wr} when the floating body vibrates in the wave of frequency ω. The wet resonant frequency can be obtained by solving the following characteristic equations

$$-\omega_{wr}^2[a_{rr} + A_{rr}(\omega)] + (c_{rr} + C_{rr} + Cm_{rr}) = 0, \tag{94}$$

where $A_{rr}(\omega)$ are diagonal elements of the added mass matrix when the floating body vibrate with the wave frequency ω. a_{rr}, c_{rr}, C_{rr} and Cm_{rr} are diagonal elements of the generalized mass matrix, the rigid matrix, the restoring matrix and the mooring force restoring matrix respectively.

Using Eq. (94), the frequency characteristics of the peaks in the curves shown in Fig. 2 may be calculated, and the results and the physical meaning of the peaks are shown in Table 3. In the table, SR and CR mean self-resonance and coupled resonance respectively. Because the range of frequency calculation is lower than 10 rad/s, the self-resonance of the modes with wet resonant frequencies higher than 10 rad/s cannot be included in Table 3. However, from the data in Table 3, it can be seen that the self resonance of three-nodes VB and one-order torsion will appear at the wave frequencies at about to 11.7 rad/s and 15.9 rad/s respectively.

From Table 3 and Fig. 2, it can be concluded that the self-resonance may be found when the wave frequencies approach the wet resonant frequencies of the modes. The wet resonant frequencies of the surge mode and the sway mode are the characteristics of the moored floating body. The self-resonance and the coupled resonance with these two modes would certainly not appear in the linear hydroelastic analysis of a free floating body (Wu, 1984; Du, 1996; Wang, 1996).

5.3. Discussion of the second-order principal coordinates

Based on the results of the generalized first order principal coordinates, the coefficient matrix, the difference frequency matrix and the sum frequency matrix in Eqs. (88) and (89) can be obtained. Thus, it is easy to obtain the generalized second-order principal coordinates of the moored floating flexible body by solving these equations. The generalized second-order difference frequency principal coordinates of the surge and the generalized second-order sum frequency principal coordinates of the two-nodes VB modes for wave angle β=180° are shown in Fig. 3 (a) and (c) respectively. The generalized second-order difference frequency principal coordinates of the sway mode and the generalized second-order sum frequency principal coordinates of the two-nodes HB modes for wave angle is β=90° are shown in Figs. 3 (b) and (d) respectively.

Figs. 3 (a) and (b) show that the resonant phenomena of the generalized second-order difference principal coordinates of the surge mode and the sway mode may be found when the difference frequencies $|\omega_{ij}^{-}|$ approach the wet resonant frequencies of the surge mode and the sway mode respectively. Furthermore, the resonant difference principal coordinates are higher when both ω_i and ω_j are smaller. Figs. 3 (c) and (d) show that the resonant phenomena of the generalized second-order sum principal coordinates of the two-nodes VB mode and two-nodes HB mode may be found when the sum frequencies ω_{ij} approach the wet resonant frequencies of the two-nodes VB mode and the two-nodes HB mode respectively.

The three-dimensional figures shown in Fig. 3 are not convenient for detailed discussion. Therefore, we also give the two-dimensional results of rigid modes in Fig. 4 where (a–f) are difference frequency principal coordinates, (g–l) are sum frequency principal coordinates and the first five flexible modes in Fig. 5 where (a–e) are differ-

Table 3
Wet resonant frequencies and the corresponding physical meanings

Serial number	Wave frequencies of the peaks		Generalized mass a_{rr}	Added mass A_{rr}	Total rigidity c_{rr}+ $C_{rr} + Cm$	Wet frequency ω_{wr} (rad/s)	Physical meaning
	ω (rad/s)	$\omega\sqrt{\frac{L}{g}}$					
r=1	0.20	0.699	9.60E+06	8.12E+05	3.99E+05	0.196	SR of surge
r=2	0.08	0.280	9.60E+06	9.24E+06	1.33E+05	0.084	SR of sway
	0.68	2.378		1.30E+07		0.077	CR with roll
r=3	0.96	3.357	9.60E+06	9.44E+06	1.76E+07	0.961	SR of heave
r=4	0.08	0.280	3.87E+08	6.86E+07	1.90E+08	0.646	CR with sway
	0.64	2.238		6.63E+07		0.647	SR of roll
r=5	0.20	0.669	8.12E+09	1.80E+10	2.26E+10	0.930	CR with surge
	1.12	3.917		1.07E+10		1.096	SR of pitch
r=6	0.16	0.560	8.27E+09	9.72E+09	3.11E+08	0.132	SR of yaw
r=7	4.16	14.55	2.37E+06	2.52E+06	8.40E+07	4.142	SR of 2-node VB
r=8	0.08	0.280	2.37E+06	1.81E+06	1.84E+08	6.622	CR with sway
	0.64	2.238		2.49E+06		6.144	CR with roll
	7.68	26.86		7.34E+05		7.687	SR of 2-nodes HB
r=9	0.20	0.699	2.33E+06	2.53E+06	5.93E+08	11.05	CR with surge
	1.12	3.917		1.99E+06		11.71	CR with pitch
r=10	0.08	0.280	1.85E+08	3.52E+07	5.55E+10	15.89	CR with sway
	1.16	4.057		3.48E+07		15.91	CR with pitch
	4.12	14.41		3.50E+07		15.90	CR with 2-nodes VB
r=11	0.16	0.560	2.33E+06	1.70E+06	1.37E+09	18.43	CR with yaw
	0.64	2.238		2.18E+06		17.44	CR with roll

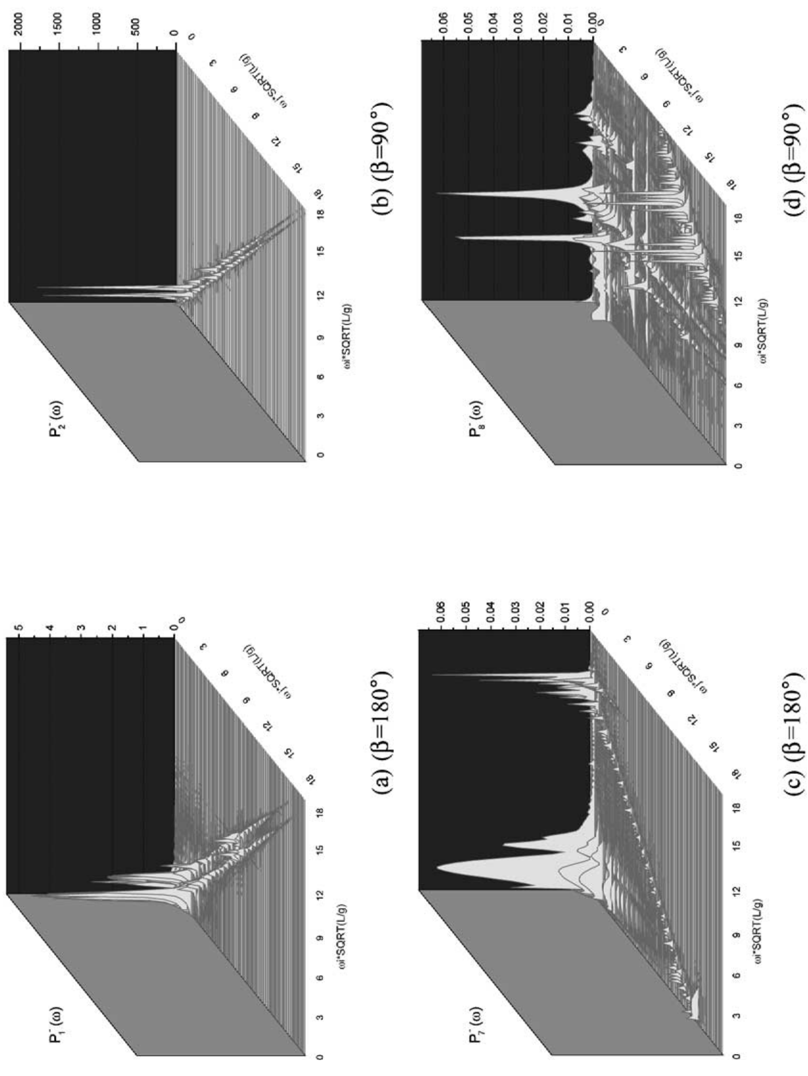


Fig. 3. The numerical results of the second order principal coordinates.

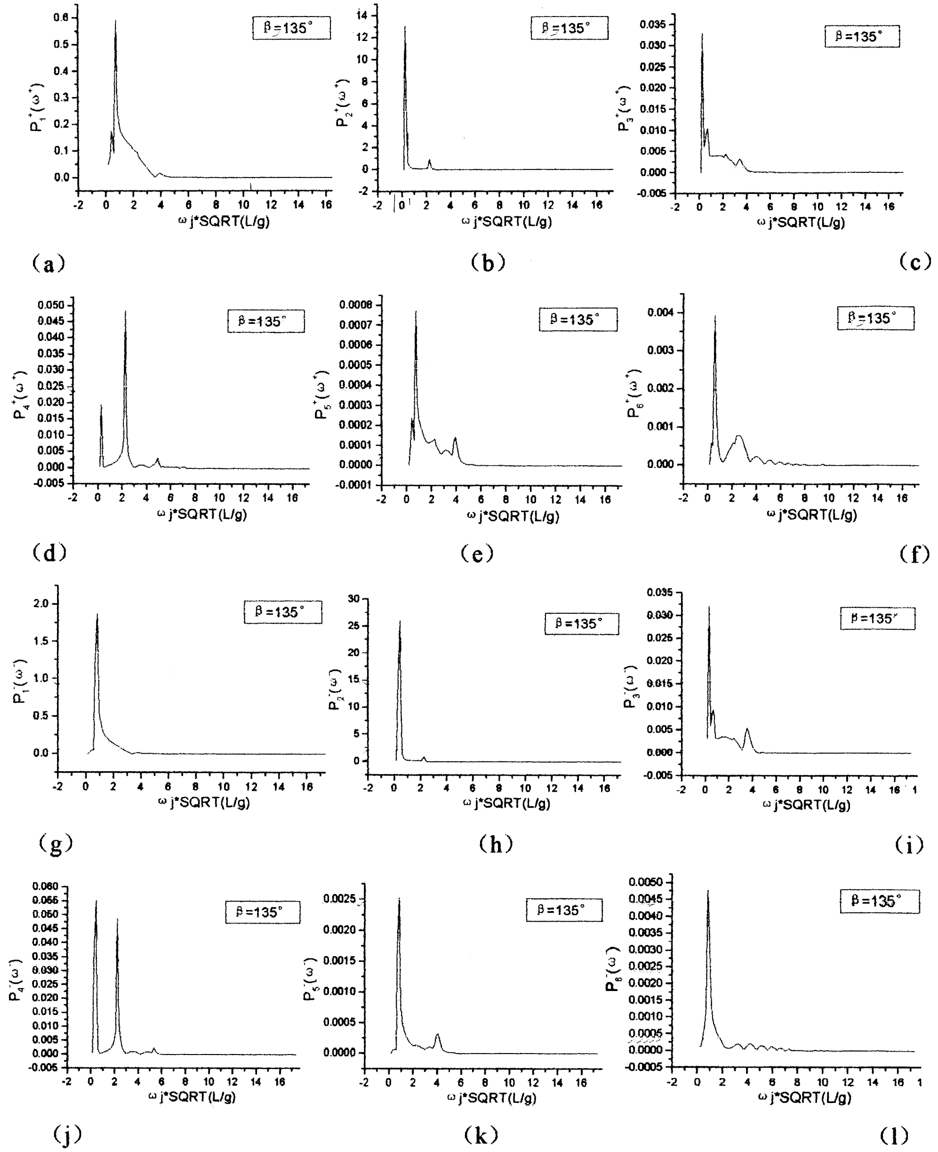


Fig. 4. The numerical results of the second order principal coordinates of rigid modes.

ence frequency principal coordinates, (f–j) are sum frequency principal coordinates. For the cases of Figs. 4 and 5, the wave angles are β=135°. The values ω_i in Figs. 4 and 5 are 0.04 rad/s and 5.00 rad/s respectively.

Figs. 4 and 5 indicate that peaks may be found in the curves of the generalized second-order principal coordinates as well as in the curves of the generalized first-order principal coordinates. The peaks and the corresponding frequencies are shown

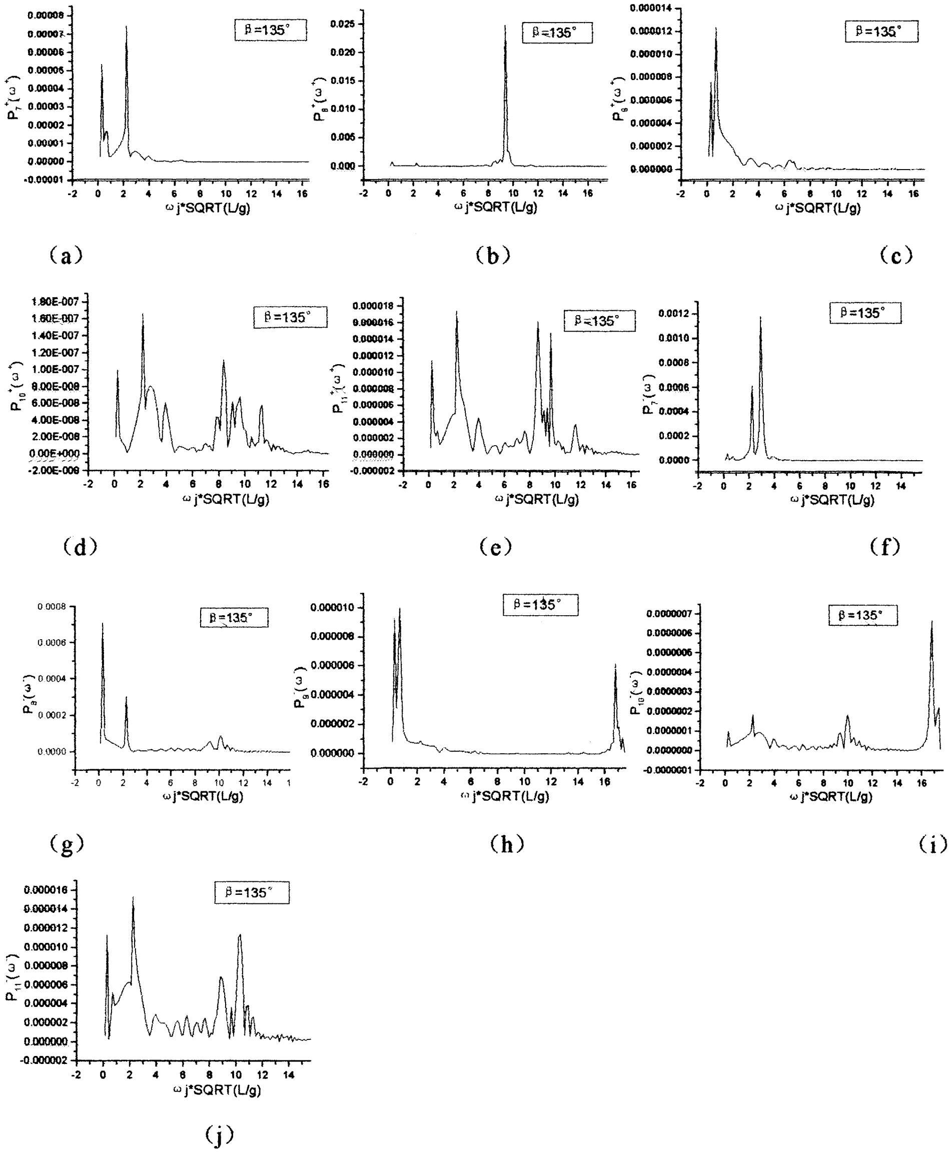


Fig. 5. The numerical results of the second order principal coordinates of the first five flexible modes.

in Table 4. Based on the discussion of the frequency characteristics of the generalized first-order principal coordinates shown in Table 3, the corresponding physical meanings of the response peaks are also given in the last column of Table 4. In this table, where SFR and DFR mean sum frequency resonance and difference frequency resonance respectively, RCB represents “resonance caused by”.

Table 4
The peaks of the second-order principal coordinates and the corresponding physical meanings

Modes	ω_{wr} rad/s	ω_i rad/s	ω_j rad/s	$\omega\sqrt{\frac{L}{g}}$	$\omega_i + \omega_j$ rad/s	$\lvert\omega_i - \omega_j\rvert$ rad/s	Physical meanings
r=1	0.196	0.04	0.12	0.419	0.16		SFR of surge
			0.20	0.699	0.24		
			0.24	0.839		0.20	DFR of surge
r=2	0.084	0.04	0.08	0.280	0.12		SFR of sway
			0.64	2.238	0.68		Couple with SFR of roll
			0.12	0.419		0.08	DFR of sway
			0.64	2.238		0.60	Couple with DFR of roll
r=3	0.961	0.04	0.08	0.280	0.12		Couple with SFR of sway
			0.20	0.699	0.24		Couple with SFR of surge
			0.64	2.238	0.68		Couple with SFR of roll
			0.96	3.357	1.00		SFR of heave
			0.08	0.280		0.04	Couple with DFR of sway
			0.20	0.699		0.16	Couple with DFR of surge
			0.68	2.378		0.64	Couple with DFR of roll
			1.00	3.497		0.96	DFR of heave
r=4	0.647	0.04	0.08	0.280	0.12		Couple with SFR of sway
			0.64	2.238	0.68		SFR of roll
			0.12	0.419		0.08	Couple with DFR of sway
			0.64	2.238		0.60	DFR of roll
r=5	1.096	0.04	0.12	0.419	0.16		Couple with SFR of surge
			0.20	0.699	0.24		
			1.12	3.917	1.16		SFR of pitch
			0.24	0.839		0.20	Couple DFR of surge
			1.16	4.057		1.12	DFR of pitch
r=6	0.132	0.04	0.08	0.280	0.12		SFR of yaw
			0.16	0.560	0.20		Couple with SFR of surge
			0.24	0.839		0.20	Couple with DFR of surge
r=7	4.142	5.00	0.08	0.280	5.08	4.92	Couple with sway RCBω_j
			0.64	2.238	5.64	4.36	Couple with roll RCBω_j
			0.84	2.937		4.16	DFR of two-nodes VB
r=8	7.687	5.00	0.08	0.280	5.08	4.92	Couple with sway RCBω_j
			0.64	2.238	5.64	4.36	Couple with roll RCB byω_j
			2.68	9.373	7.68		SFR of two-nodes HB
r=9	11.71	5.00	0.08	0.280	5.08	4.92	Couple with sway RCBω_j
			0.16	0.560	5.16	4.84	Couple with surge RCBω_j
			4.80	16.79		0.20	Couple with DFR of surge
r=10	15.90	5.0	0.08	0.280	5.08	4.92	Couple with sway RCBω_j
			0.64	2.238	5.64	4.36	Couple with roll RCBω_j
			2.40	8.394	7.40		Couple with SFR of 2-node HB
			4.80	16.79		0.20	Couple with DFR of surge
			4.96	17.35		0.04	Couple with DFR of sway
r=11	17.44	5.0	0.08	0.280	5.08	4.92	Couple with sway RCBω_j
			0.64	2.238	5.64	4.36	Couple with roll RCBω_j
			2.48	8.674	7.48		Couple with SFR of 2-node HB
			2.76	9.653	7.76		

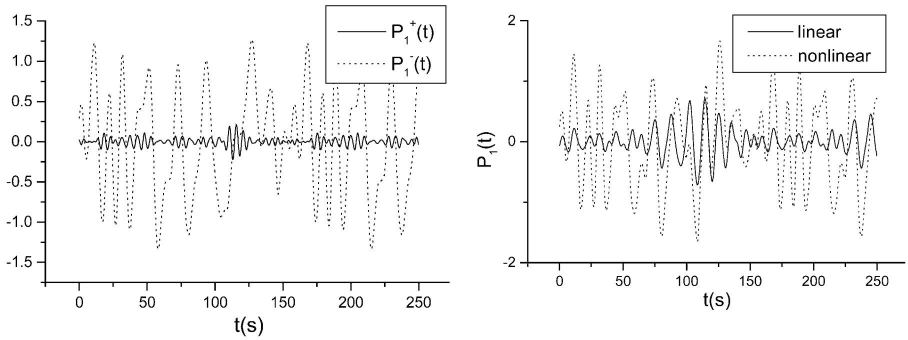


Fig. 6. Time history of the surge principal coordinate.

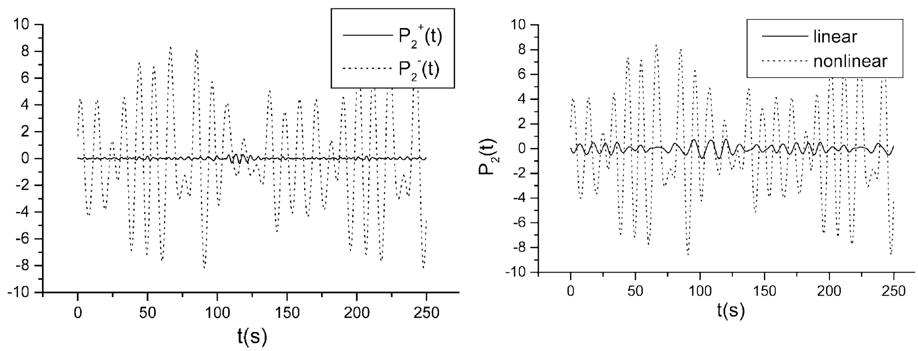


Fig. 7. Time history of the sway principal coordinate.

5.4. Discussion of the principal coordinate responses

Substituting the principal coordinates into Eqs. (90–92) and the Eq. (80), the resultant principal coordinate responses are obtained. The numerical results for the wave angle β=135° are shown in Figs. 6–20. In order to investigate the contributions from

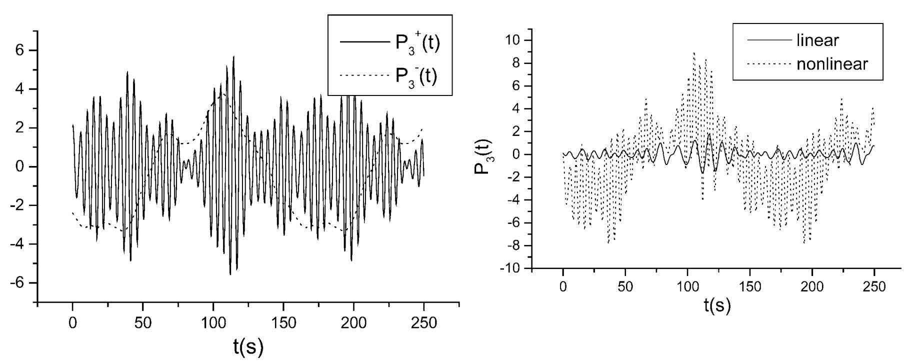


Fig. 8. Time history of the heave principal coordinate.

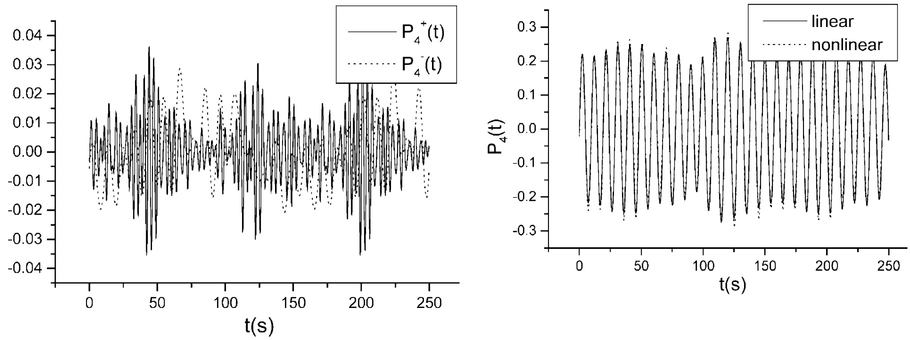


Fig. 9. Time history of the roll principal coordinate.

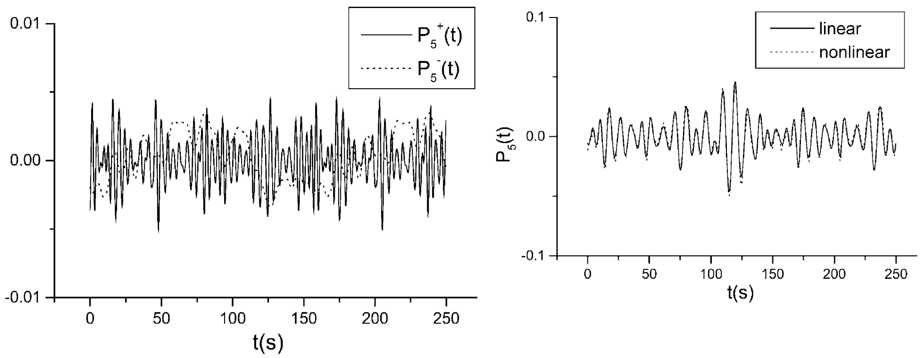


Fig. 10. Time history of the pitch principal coordinate.

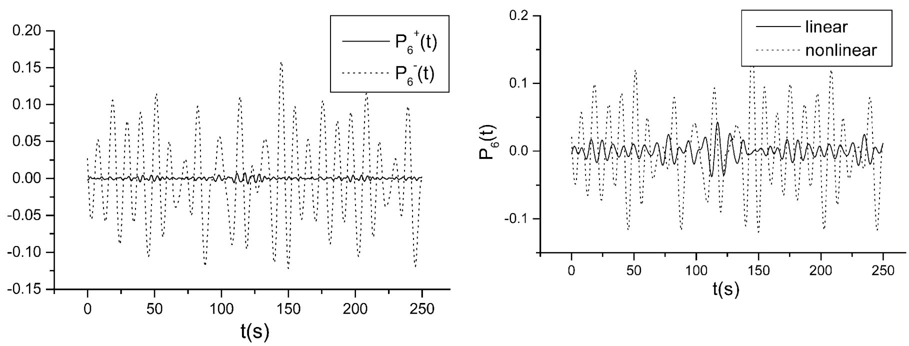


Fig. 11. Time history of the yaw principal coordinate.

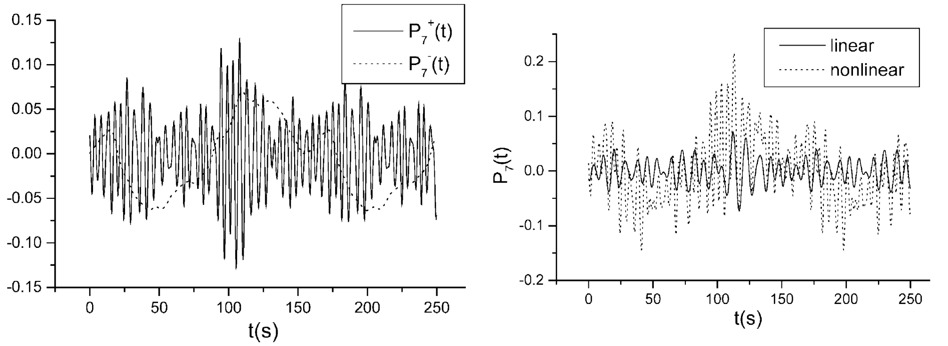


Fig. 12. Time history of the principal coordinate of the two-node VB mode.

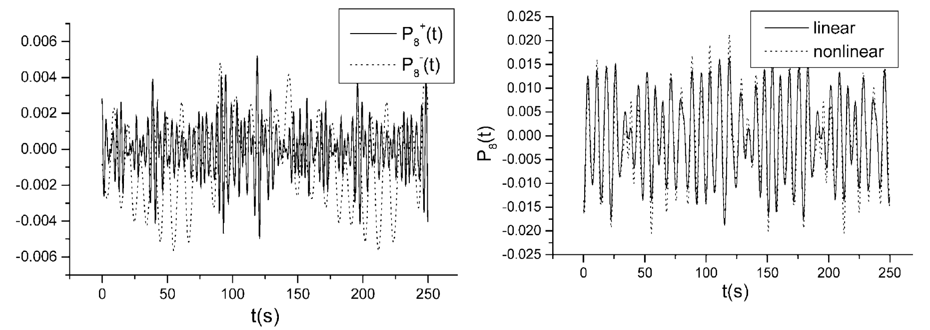


Fig. 13. Time history of the principal coordinate of the two-node HB mode.

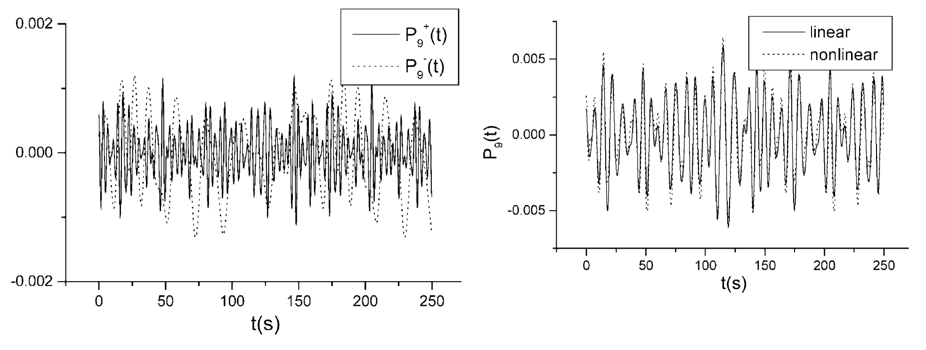


Fig. 14. Time history of the principal coordinate of the 3-node VB mode.

difference frequencies and sum frequencies, both of them are shown in the same figure on the left side. The comparisons of the linear and nonlinear responses are shown in the same figures of Figs. 6–20 on the right hand side. In these figures, “nonlinear” represents the total responses including the linear and nonlinear contributions.

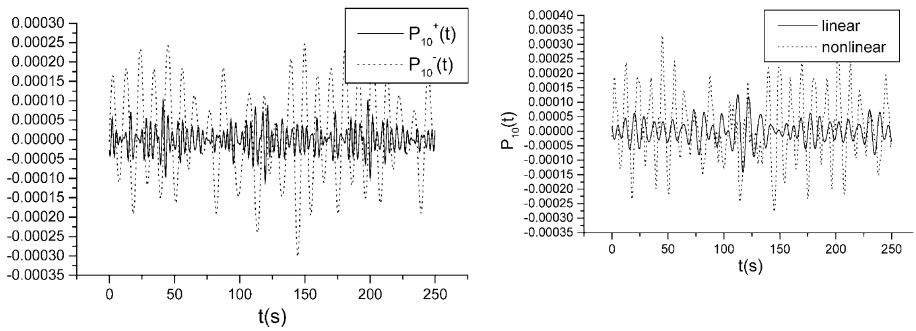


Fig. 15. Time history of the principal coordinate of the one-node torsion mode.

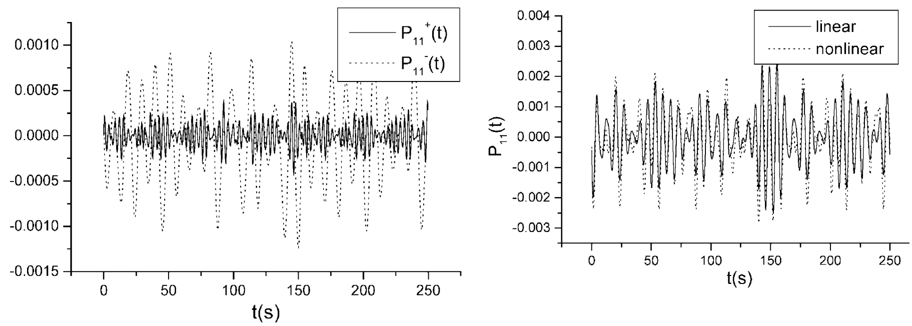


Fig. 16. Time history of the principal coordinate of the 3-node HB mode.

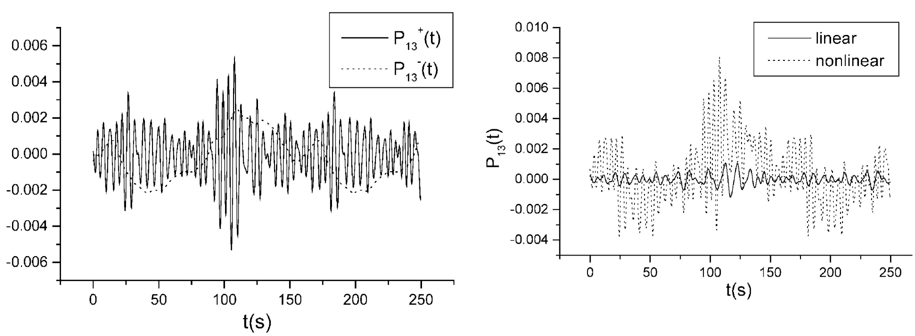


Fig. 17. Time history of the principal coordinate of the 4-node VB mode.

From these calculations, the following conclusions can be drawn:

1. In general, the influences of the second-order response on the principal total responses of resultant principal coordinates have the following characteristics: (a) when the difference frequency response amplitudes are bigger than those of the

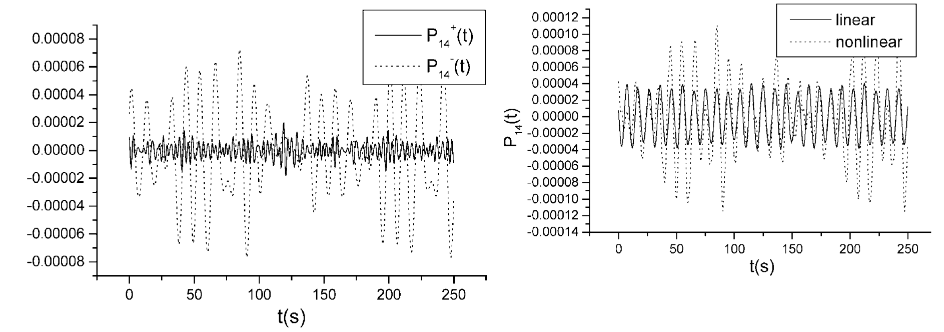


Fig. 18. Time history of the principal coordinate of the two-node torsion mode torsion.

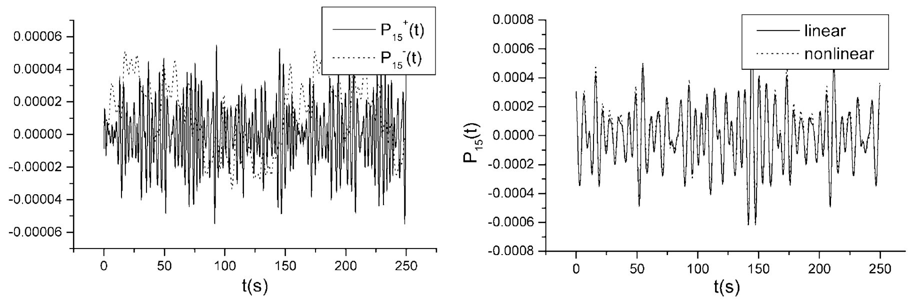


Fig. 19. Time history of the principal coordinate of the 4-node HB mode.

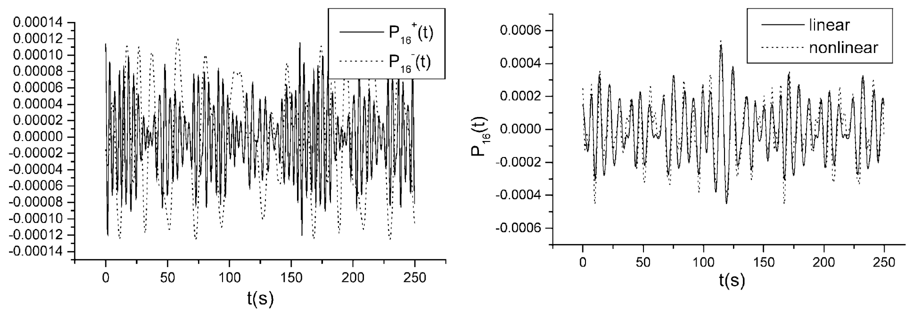


Fig. 20. Time history of the principal coordinate of the 5-node VB mode.

sum frequency responses, the total responses including the contribution of the second-order nonlinear forces will be larger than the linear ones [Figs. 6, 7, 11, 15 and 18]; (b) when the difference between them is not big and when the difference frequency is small, the apparent slow varying phenomenon of the difference frequency principal coordinates will appear and the total responses including the contribution of the second-order nonlinear forces will be larger than the linear

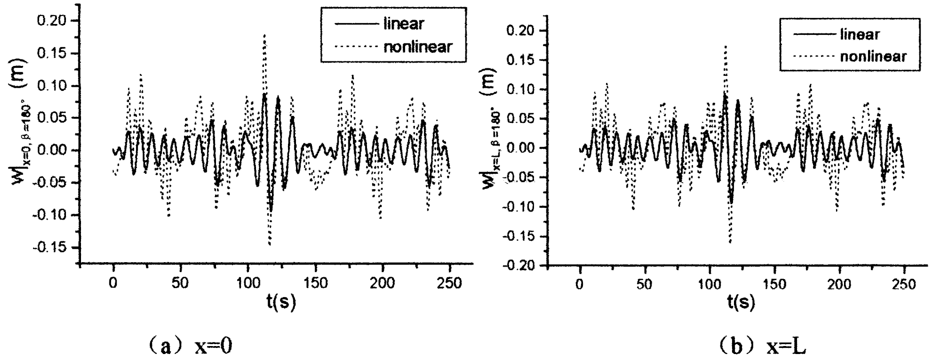


Fig. 21. Vertical displacement at the bow.

ones [Figs. 8, 12 and 17]; (c) when the ascendant frequencies of difference frequency response and the sum frequency reponse are of the same order of magnitude, the effects of the second-order forces on the total responses is small [Figs. 9, 10, 13, 14, 16, 19 and 20].

2. At the wave angle β=135°, the total principal coordinates of the surge mode [Fig. 6], sway mode [Fig. 7], heave mode [Fig. 8], yaw mode [Fig. 11], two-node vertical bending mode [Fig. 12], one-node torsion mode [Fig. 15], four-node vertical bending mode [Fig. 17] and the two-node torsion mode [Fig. 18] are moderately affected by the second-order forces, the slow varying phenomenon appears in the total principal coordinate responses of the heave mode, two-node vertical bending mode and four-node vertical bending mode, the period of the varying is about 157 sec, and the responses of the roll mode [Fig. 9], pitch mode [Fig. 10], all horizontal bending modes [Figs. 13, 16 and 19], three-node mode and the five-node vertical bending mode [Figs. 14 and 20] are little affected by the second-order forces.
3. As is well known, the surge mode, sway mode and the yaw mode motions will be greatly affected by the nonlinear wave drift forces. The numerical results of these modes presented above are in agreement with this conclusion. For other modes, the following phenomena can be observed. (a) The fore-after and port-starboard symmetric modes such as heave, two-node vertical bending, four-node vertical bending and one-node axial tension modes are more affected by the second-order forces. The reason may be that the energy of these modes is not easy to be dissipated because the motions of these modes are not coupled with the surge, sway or yaw modes. (b) The port-starboard symmetric and fore-after antisymmetric modes such as the pitch, three-node vertical bending and the five-node vertical bending modes are little affected by the second-order forces because the energy of these modes is easily transformed into surge motion by the coupling vibration. (c) The fore-after symmetric and port-starboard antisymmetric modes such as the roll, two-node horizontal bending and the four-node horizontal bending modes are little affected by the second-order forces. The reason may be that the energy of these modes is easy to be transformed to the vibration of coupling sway

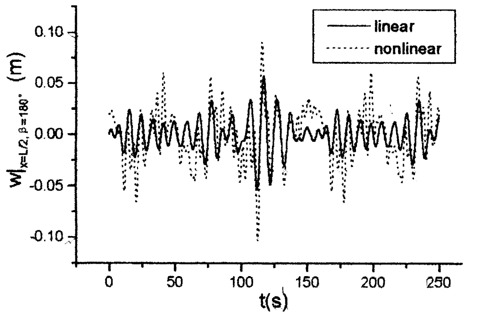


Fig. 22. Vertical displacement at x =L/2.

mode, which has the same symmetry characteristics. (d) For port-starboard and fore-after symmetric modes, the second-order forces have little effect on the three-node mode because it has coupled vibration with the yaw mode, which has the same symmetry characteristics; the one-node torsion mode and the two-node torsion mode are affected by the second-order forces. The reason may be that these modes have no coupled vibration with the yaw mode even the symmetry characteristics of them are the same.

5.5. Responses of displacements, bending moments and shearing forces

For practical application, the vertical displacements, vertical bending moments and the vertical shearing forces are of more interest. The numerical results of the vertical displacements are shown in Figs. 21 and 22. The numerical results for the vertical bending moments and the vertical shearing forces are shown in Figs. 23 and 24 respectively.

From these figures, it can be found that the influences of the second-order forces on the vertical displacements, bending moments and the vertical shearing forces do exist. The vertical displacements of the bow are much higher than that at the middle length of the floating beam. Because of the antisymmetry of the action of the wave,

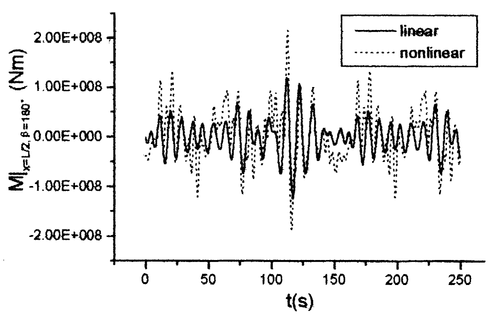
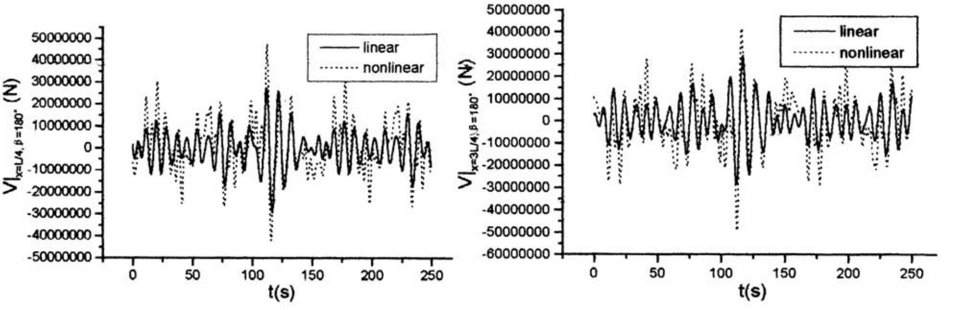


Fig. 23. Vertical bending moment at x=L/2.



(a) $x = L/4$ (b) $x = 3L/4$

Fig. 24. Vertical shearing force.

the absolute values of the vertical shearing forces at one-quarter length of the floating beam are slightly bigger than that of the three-fourth length of the floating beam.

6. Conclusions

The hydroelasticity theory provides a more rational and more consistent approach for the assessment of the dynamic responses of a flexible floating structure in waves. The linear hydroelasticity theory has been well developed now but few dealing with the nonlinearity which is an important factor for many practical applications.

By integration of the second-order fluid pressure over the instantaneous wetted surface, the generalized first- and second-order fluid forces needed in nonlinear hydroelastic analysis have been obtained. The generalized first order wave exciting forces, radiation forces and restoring forces have the same form as in the linear hydroelasticity theory. The generalized second-order fluid forces consist of five parts: the generalized second-order wave exciting forces $F_r^{(2)}(t)$ which are related to the incident and diffracted wave potentials; the generalized second-order radiation forces $D_r^{(2)}(t)$ related to the radiation potentials; the generalized second-order forces $E_r^{(2)}(t)$ induced by the coupling between radiation wave potentials and the incident-diffracted wave potentials; the generalized second-order forces $S_r^{(2)}(t)$ induced by the movement of wetted surface; and the generalized second-order forces $\Delta Z_r^{(2)}(t)$ induced by the instantaneous variation of the wetted surface.

Based on these, the nonlinear hydroelastic equations for arbitrary floating bodies in irregular waves has been presented, and the solution method has been discussed. By employing the theory of irregular waves being described by the aggregation of regular waves, the expressions of the coefficients of the generalized first- and second-order fluid forces to irregular waves in the equilibrium frame have been deduced. The linear and nonlinear three-dimensional hydroelastic equations of a moored floating system in frequency domain have been established. The solution of the total principal coordinates responses $p_k(t)$ consist of the steady components due to the excitation of $Z_r^{(0)}$ and Q_r, the wave frequency components due to the excitation from the terms of D_{r0j}, the difference frequency components coming from the excitation terms of Q_{rij} and the sum frequency components coming from those of D_{rij}. Some numerical results are given to demonstrate the validity of the theory using a moored box-type floating barge as an example. Some conclusions based on the calculated results are drawn.

Appendix A

The coefficients in the expression of the generalized second-order forces induced by the instantaneous variation of the wetted surface can be expressed as

$$K_r^*(\omega_i,\omega_j) = -\frac{\rho}{4g}\int_{C_W} \{(\vec{n}\cdot\vec{u}_r^{\,0})(i\omega_i)[\varphi_I(\omega_i) + \varphi_D(\omega_i)]\,(-i\omega_j)[\bar{\varphi}_I(\omega_j) + \bar{\varphi}_D(\omega_j)]\}\frac{d\,l}{\sqrt{1-n_3^2}}, \tag{A1}$$

$$K_r(\omega_i,\omega_j) = -\frac{\rho}{4g}\int_{C_W} \{(\vec{n}\cdot\vec{u}_r^{\,0})(i\omega_i)[\varphi_I(\omega_i) + \varphi_D(\omega_i)]\cdot(i\omega_j)[\varphi_I(\omega_j) + \varphi_D(\omega_j)]\}\frac{d\,l}{\sqrt{1-n_3^2}}, \tag{A2}$$

$$\bar{K}_{rk}(\omega_i,\omega_j) = -\frac{\rho}{4g}\int_{C_W} \{(\vec{n}\cdot\vec{u}_r^{\,0})(i\omega_i)[\varphi_I(\omega_i) + \varphi_D(\omega_i)]\cdot[(-i\omega_j)\bar{\varphi}_k(\omega_j) - gw_k^0]\}\frac{d\,l}{\sqrt{1-n_3^2}}, \tag{A3}$$

$$K_{rk}(\omega_i,\omega_j) = -\frac{\rho}{4g}\int_{C_W} \{(\vec{n}\cdot\vec{u}_r^{\,0})(-i\omega_i)[\bar{\varphi}_I(\omega_j) + \bar{\varphi}_D(\omega_j)]\cdot[(i\omega_i)\varphi_k(\omega_i) - gw_k^0]\}\frac{d\,l}{\sqrt{1-n_3^2}}, \tag{A4}$$

$$K_{rk}^*(\omega_i,\omega_j) = -\frac{\rho}{2g}\int_{C_W} \{(\vec{n}\cdot\vec{u}_r^{\,0})(i\omega_i)[\varphi_I(\omega_i) + \varphi_D(\omega_i)]\cdot[(i\omega_j)\varphi_k(\omega_j) - gw_k^0]\}\frac{d\,l}{\sqrt{1-n_3^2}}, \tag{A5}$$

$$G^*_{rkl}(\omega_i,\omega_j) = -\frac{\rho}{4g}\int_{C_W} \{(\overrightarrow{n}\cdot\overrightarrow{u}\,^0_r)[(i\omega_i)\varphi_k(\omega_i)-gw^0_k]\cdot[(-i\omega_j)\bar{\varphi}_l(\omega_j) \tag{A6}$$

$$-gw^0_l]\}\frac{d\,l}{\sqrt{1-n_3^2}},$$

$$G_{rkl}(\omega_i,\omega_j) = -\frac{\rho}{4g}\int_{C_W} \{(\overrightarrow{n}\cdot\overrightarrow{u}\,^0_r)[(i\omega_i)\varphi_k(\omega_i)-gw^0_k]\cdot[(i\omega_j)\varphi_l(\omega_j) \tag{A7}$$

$$-gw^0_l]\}\frac{d\,l}{\sqrt{1-n_3^2}}.$$

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